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THESIS

EFFECTIVENESS OF A MINE AVOIDANCE SENSOR IN MINEFIELD TRANSIT

by

Eng Yee Toh

March 2005

Thesis Advisor: Steven E. Pilnick
Co-advisors: Donald P. Gaver
Patricia A Jacobs

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EFFECTIVENESS OF A MINE-AVOIDANCE SENSOR ON MINEFIELD TRANSIT

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Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

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TABLE OF CONTENTS

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LIST OF FIGURES

LIST OF TABLES

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EXECUTIVE SUMMARY

The concept of Mine Avoidance is an attractive alternative to traditional mine clearance, and mine avoidance sonars are increasingly being promoted as valuable assets in mine-counter-measures warfare. The mine avoidance sensor (MAS) is able to detect mine-like objects but currently *cannot classify the objects*. As with any new technology, there are limits to what mine avoidance systems can do. This thesis uses simulation to study the effectiveness of the mine avoidance sensor on minefield transit.

Previous research in this area showed that mine-avoidance is tenable for certain general environmental conditions, but beyond that mine avoidance may not be effective because of the high clutter density of mine-like objects in the area. The minefield transit models (Simple Minefield Transit, Minefield Objects Avoidance Maneuvers (MOAM)) investigated were using minefields with infinite width and finite length across. (Kim, 2002), and indicated that maneuver and avoidance tactics have a considerable influence on the outcome of the transit.

This thesis builds upon the previous research, and explores further the Neighboring Diagonal Local Avoidance Model (NDLAM) analytical model developed by Gaver *et al* (Gaver, Jacobs and Pilnick 2003) on the discrete version of the MOAM model explored in Kim's thesis. The aim is to introduce more realism to the research and study how a more sophisticated maneuver would affect the outcome. This simulation introduces three new simulation models: firstly, the discrete minefield version of the NDLAM model that is used to provide validation for the later models; second, a simulation model for another analytical model to validate the forward checking portions of the new maneuver model; finally the Enhanced Maneuver Model (EMM), which implements a more sophisticated maneuver model to mimic real maneuvers in MCM operations. The simulation results from the EMM model are used to study the effectiveness of the mine avoidance sensors and suggest possible mine avoidance tactics.

In all three simulation models, the minefield is finite, i.e. of fixed width and length, and is represented by a grid, similar to the representation of a minefield in MCM

ships. The MCM vessel would typically go ahead, left-forward, right-forward, left, right or astern when taking avoidance maneuvers. The distances traveled are influenced by the detection range of the sonar system. The simulation tiles the space with squares. The size of the squares is specified by the typical detection range of the sonar which is assumed to be greater than the actuation mine radius of about 100 meters. Each square in this grid either contains a mine, a non-mine-mine-like bottom object (NOMBO) or is empty; these events are mutually exclusive, i.e., a square cannot contain both a mine and NOMBO in this model. Each square contains at most one object. The minefield has fixed length L and width W and therefore has a finite area.

A ship's maneuver is simplified in that a platform "moves" in every time step of the simulation, may or may not detect objects in an adjacent square, and makes decisions according to a rule set on where to turn if an object is detected in the square in its path. This maneuver can be easily represented on the screen. The representation of the ship maneuvering in the simulation is similar to that of a wall tracing algorithm for a computer mouse going through a maze.

Simulation experiments are conducted with the mines and NOMBOs sown into the minefield in a manner approximating a spatial Poisson process. First, with specified rates of mine density and NOMBO density for the whole area, random numbers drawn from a Poisson distribution are used to represent the actual number of mines and NOMBOs in the minefield . With the numbers of the objects obtained for each type, each object will have a (row, column) coordinate that is obtained from randomly drawn numbers from the specified rows and columns. If the square corresponding to the coordinate is empty, then an object is "planted" there, and the attributes of the square changed to reflect that there is now an object in the square. Mines will be planted first, followed by NOMBOs. If a square has been occupied already, then new coordinates are drawn. There can be zero, one mine <u>or</u> one NOMBO in any square. This object-sowing process will repeat until all the mines and NOMBOs are planted. Experiments are conducted with the Spatial Poisson distribution at first, and for subsequent simulations, the number of mines and NOMBOs are simulated using Binomial distributions.

The simulation of the minefield traversal was coded in Java, and is based on a step-based simulation because it allows maneuvers through the grid to be studied and visualized. The simulation results indicated that the outcomes are very similar whether a Spatial Poisson or Binomial Distribution was used for the distribution of the mines and objects.

The detection of objects (mines and NOMBOs) is simulated by first checking if a square ahead is occupied by a NOMBO or mine. If it is, then a random number is drawn to compare against the prevalent probability of detection, pd, used. If the number drawn is smaller than pd, then the object is detected and marked as detected and avoidance actions taken.

In the discrete NDLAM model, the vessel checks the front three squares ahead sequentially to see if there is a mine or NOMBO. If no objects are detected, the vessel would move forward and repeat the whole process until it reaches the end of the minefield or encounters an undetected mine and is killed. If all three squares forward are blocked, the vessel backtracks all the way to the initial starting position and restarts a new transit attempt that does not intersect any previous path; this is possible because the minefield in the NDLAM model is of infinite width. In this simulation, a new minefield is redrawn independently each time the ship returns to the beginning of the minefield using the same expected mine and NOMBO density. It was found that the results obtained are similar to the analytical computations, and indicate that the models are consistent with each other. The model also provides some validation basis for the backtracking portions for the EMM.

The second model was developed as a special case for the Enhanced Maneuver Model (Jacobs, 2005) to validate the forward checking portions. The simulation model comprises a special 3-square wide minefield with fixed length L. The vessel checks all the squares ahead as in the discrete NDLAM model and also the left and right squares. However, it does not backtrack if all sides are blocked, and the attempt is considered a failure due to blockages. In this model it is also possible to have a failure due to an encounter with an undetected mine. The simulation results are similar to the analytical computations, indicating that the simulation model was consistent with the analytical model.

The 3rd model, Enhanced Maneuver Model (EMM) builds upon the first two models: it checks all the squares ahead and to the sides as in the special cases, but also backtracks 1 square each time if all the front squares are blocked to check the sides further. The measure of performance is the probability of successful transit. There are three outcomes: success, failure due to undetected mine detonations, or failure due to blocked paths. Due to the complexity of the model, an analytical version has yet to be developed, so it relies on the validation of special cases from the two previous models. More realistic mine and NOMBO densities are used in the EMM simulation.

.The NDLAM model has an infinite minefield and the EMM model has a finite minefield. In the NDLAM model, the possible outcomes are either mission success or failure (vessel/platform kill) due to encountering an undetected mine and is killed. As the vessel can always backtrack and restart in a different position so that it will not intersect a previous path, there will always be an outcome where the vessel will go through the minefield, or will encounter an undetected mine. In EMM, the outcomes were success, failure due to encounter with an undetected mine, or failure due to blocked paths. In the study by Kim, it was discovered that there was a phenomenon that the probability of safe transit actually decreases when pd increases, which is an apparent paradox as one would expect the probability of safe transit would increase when better sensor capability exists. However, as pd becomes better, the vessel detects more objects and thus takes more avoidance actions, resulting in more time and distance spent in the field, thus influencing the probability of successful transit downwards since the exposure to undetected mines becomes higher. Results similar to Kim's, demonstrating the mine avoidance paradox, were observed in this research with the discrete minefield when using the maneuvering rules of the NDLAM model with all of its backtracking. However, for the same object densities, the enhanced maneuvering rules of the new EMM model resulted in steadily improving probability of successful minefield transit as sensor Pd increases. *It was observed that for these parameter values the more sophisticated object avoidance maneuvering mitigates the mine avoidance paradox.* Further investigation with the EMM model found that degradation of probability of successful transit does occur with extremely high NOMBO densities and high Pd. But in the cases discovered the degradation was due to blocked paths across the finite size minefield rather than undetected mine detonations.

Based on the NOMBO and mine density alone, the results from this thesis indicate that the probability of successful minefield transit is increased with the use of mine avoidance sonars when the mine and NOMBO density is lower than half the area of the minefield when using the EMM maneuvers, assuming other factors are held constant. It must be noted that the assumptions for the EMM maneuvers are that the vessel is equipped with accurate positioning systems that allows it to demarcate positions of objects correctly and that it is able to navigate accurately and correctly without endangering itself along the way. The probability of detection of mines and NOMBOs are also assumed to be the same. Other factors such as the capability of the mine, other environmental factors that affect the probability of detection are not studied here, and could be researched further.

The results obtained from the thesis could form the basis of some mine avoidance tactics with regards to open waters, coastal areas, harbors and Q-route surveys. Some suggestions are made on the usefulness of the MAS system with regards to the prevalent object density. In general when the total object density is less than half of the area to be transited, the EMM maneuvers would be useful and provide higher probability of mission success with the use of MAS. However, once the object density increases beyond half of the minefield, the probability of success will decrease due to increased blockages for the parameters studied. The results also show that the ability to classify objects as mines or NOMBOs would play an important part in improving the mine-avoidance systems. These ideas can be used for either MCM or mine warfare to improve or curb the usefulness of the MAS. In summary, the MAS is effective in enhancing probability of safe transit as pd increases, but the technology is not mature yet to completely replace traditional MCM systems.

I. INTRODUCTION

A. BACKGROUND

Naval mine counter-measure (MCM) efforts are integral to modern naval warfare, be it wide-area mine clearance to clear approach lanes in preparation for amphibious landings; or, to clear dangerous mined waters for a carrier battle group to pass through unscathed; or, to simply conduct mine reconnaissance missions along important Sea Lanes of Communications (SLOCs) to ensure the seas are mine-free for the safe transit of friendly naval combatants and commercial vessels. However, due to difficulties in underwater operations with varying environmental conditions and system performances, and the sheer size of area to be covered, MCM operations are inherently time-consuming and expensive. Typical clearance operations, for example, minesweeping or minehunting, take a disproportionate time to neutralize mine threats, and in scenarios where expediency is of the essence for a naval force to transit a mined area, force deployments may not afford the time for completely thorough MCM operations to be carried out for the safe transit of the naval combatants. Also, due to the dynamism of naval warfare, MCM assets may not arrive in the theatre in time to conduct the operations, thus hampering the conduct of other operations crucial to the objective.

Thus, new ways are continually being explored to expedite the MCM process. Going back to first principles, the best tactic for MCM is actually to avoid possibly lethal components of the minefield altogether whenever possible. The advent of organic mine-avoidance sonars (MAS) offers such a possibility. The aim is for naval combatants or auxilliaries equipped with such systems to be provided with protection against sea mines, by avoiding areas with possible mines allowing the sensor platform to transit safely, without having to wait for the completion or the conduct of mine-clearance operations. Mine sweeping and mine hunting are not perfect. MAS can also be used in mine field areas that have been incompletely swept.

1. Concept of Mine Avoidance

The idea is simple: instead of waiting for MCM assets to clear a path, a vessel (ship and/or submarine) equipped with a forward-looking sonar system could potentially transit the minefield safely if it knows where the suspicious objects are and maneuvers to avoid those objects. The sonar, typically installed under the bow of the ship, would be used to scan the waters ahead as the ship moves so that minelike objects that are detected ahead would be marked and avoided. This is analogous to a blind person (for the ship is "blind" underwater) holding and sweeping his guide stick on the road ahead when he walks, avoiding pot holes, fire hydrants or other obstructions along the way if his stick detects their presence.

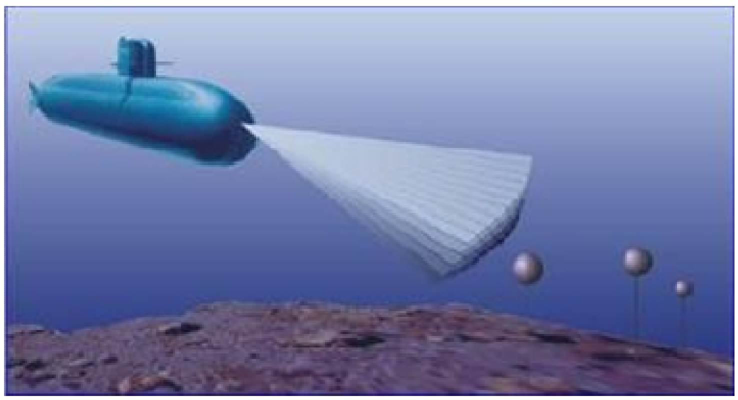

Figure 1. Example of a Mine Avoidance Sonar installed on a submarine (from L3 Communications: SCOUT Mine Detection and Obstacle Avoidance Sonar NDS 3070 for Submarines)

2. Challenges for Mine Avoidance

The concept of mine avoidance has been heavily promoted as the next wave of MCM development, but as with any new technology, there are pitfalls or limits to what can be achieved with a MAS. The sonar has a probability of detection less than 1 and can also generate false alarms (reports an object when there is none). With the current state of technology, the sonar is able to classify an object as either minelike or non-minelike, *but is not able to differentiate between a mine and a non-mine mine-like bottom object (NOMBO).* For example using the blind person analogy above, most of the time, the blind person would be able to avoid most obstacles, and find a way around them, but sometimes the stick may not detect the obstacle in time and he may walk or bump into it. Even if he/she is able to detect something in front, he/she would not able to "identify" what type of obstacle this is unless he/she can physically examine the obstacle and is familiar with its shape to know whether it would trip him/her or not.

Similarly, a sonar system may not detect underwater objects in time or miss them completely, and thus traverse over the objects. Where the blind person may trip or fall or get dirty if he fell into a puddle, the ship may be damaged or killed if the object the ship transits over is a mine and it detonates. Also, if there are sufficient quantities of obstacles in the seabed, and the vessel is not able to differentiate between a mine and a non-mine mine-like object, the vessel would attempt to conservatively avoid all the detected objects, and thus travel a larger path within the minefield. Traveling a larger path within this minefield results in more chances to encounter mines, some of which may not be detected and thus sink the ship. The ship can also fail to get across the minefield because it may run out of maneuvering space. In such a case, it would not be able to cross the minefield successfully without having to go over some detected objects; if the object is a mine, it may detonate and kill the ship. Therefore, the likelihood of a successful transit through a mined area depends on several factors including the density of underwater minelike objects, the successful detections of objects ahead, and the way the ship maneuvers when objects are detected. Thus, the resultant problem lends itself to a study of the effectiveness of a mine avoidance sonar when examined in conjunction with varying parameters of obstacle density, sensor performance (influencing probability of detection) and also vessel maneuvers.

B. PREVIOUS RESEARCH

1. Minefield Objects Avoidance Maneuver Model (MOAM)

Previous research in this area showed that mine-avoidance is tenable for certain general environmental conditions, but beyond that mine avoidance may not be effective because of the high clutter density in the area. (Kim, 2002). Kim used simulation and corresponding analytical models to obtain results concerning the probability of safe minefield transit and the distance traveled in the minefield. It was found that the probability of safe minefield transit is highly dependent on the rate of occurrence of mines and non-mine, minelike bottom objects (NOMBOs) in the minefield. The minefield transit models investigated (Kim, 2002) indicated that representation of maneuver and avoidance tactics in the simulation have a considerable influence on the outcome of the transit. The sensor’s inability to differentiate between sea mines and NOMBOs is also a serious limitation in the conduct of mine-avoidance operations.

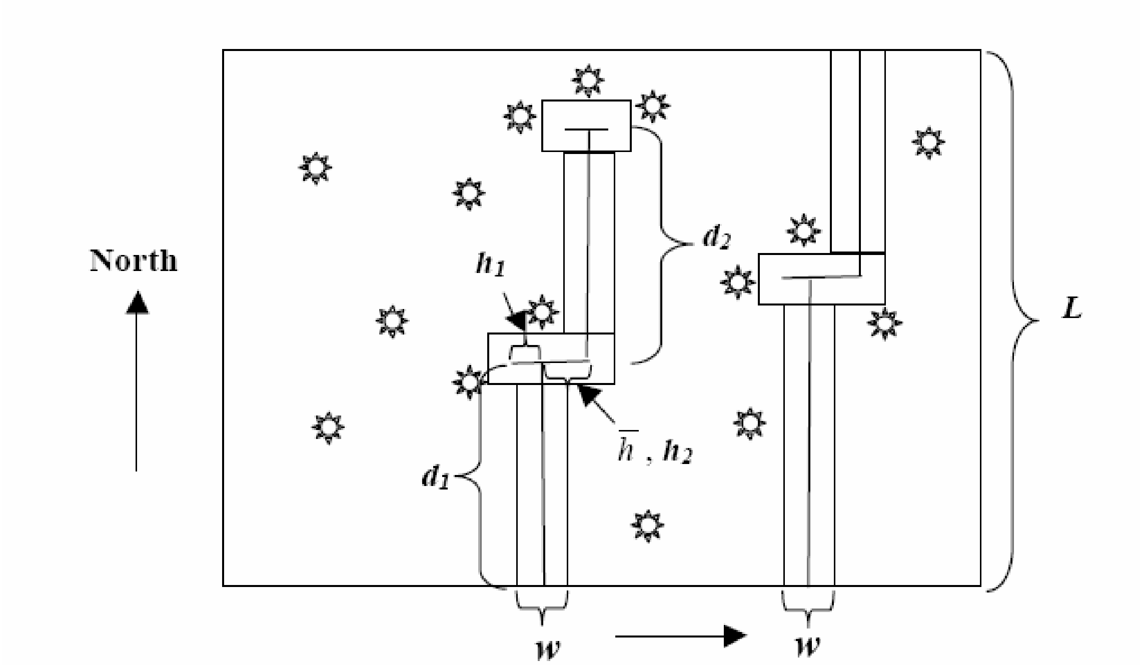

Figure 2. Pictorial Representation Of Minefield Object Avoidance Maneuver (MOAM) Analytical Model (From: Kim, 2002)

2. Model and Simulation with Continuous Probability

In the two models explored in (Kim, 2002), corresponding analytical models were constructed, which allowed the calculation the probability of safe minefield transit mathematically. He then conducted simulations using a spatial Poisson process model for the location of objects. He used the analytical model to verify the simulation results were statistically equal to those of the analytical model. The simulation was then used to estimate additional MOPs. The first model used was a simple object-avoidance model in which the distance to encounter a mine, NOMBO or false alarm is simulated by drawing independent exponential random numbers to represent the various distances to the first mine, first NOMBO, and first false alarm . If the minimum of the distances drawn is larger than the distance to be traveled across the minefield, L, this means that the ship has transited safely through the minefield without encountering any mines, NOMBOs or false alarms. If any of the simulated distances to the first mine, NOMBO or false alarm drawn is less than L, then it means that the ship has encountered one of them; if the ship has encountered an object, a second random number is drawn to compare against the prevalent probability of detection of that object to determine if that object is detected. If it is not detected, then the vessel will either blow up if the object is a mine, or will safely pass if the object is a NOMBO or false alarm. If the object is detected, then the vessel would backtrack to a new starting point and attempt transit along a nonintersecting path, until the vessel passes successfully through the field or is blown up by a mine. The second model, the Minefield Object Avoidance Maneuver model, is a more sophisticated model where the avoidance maneuvers are slightly more complex. The probability of safe passage through the field results obtained from the second model were as expected, larger than those for the first model, as the extra maneuvers enable the ship to transit the minefield with higher probability of success. A subsequent research of similar nature also lends some support to the finding that representation of maneuvers in a simulation model affects the probability of mission success. (Nawara, 2003)

Figure 3. Example Of An Organic Shipboard Mine Avoidance Sonar (From Stn Atlas Elektronik Gmbh - Mine And Obstacle Avoidance Sonar Mas 90)

C. AIM AND STRUCTURE OF THESIS

Building up on what has been completed, this research will explore and evaluate the effectiveness of generic ship-borne mine-avoidance sensors (MAS) in enabling safe minefield transit for the platforms carrying the sensors. The minefield representation will be to "tile" a finite area with disjoint squares. Each square independently either contains an object or not. This study will investigate the results of using mine avoidance in this discrete minefield model with a more complex maneuver. The principal measure of effectiveness is the probability of safe minefield transit. Chapter II contains details of this discrete minefield model as presented by Gaver *et al* 2003. Chapter III describes how objects and false alarms are modeled and describes the algorithm used to represent maneuver through the minefield. Chapter IV presents the results from the simulation and compares it to the results derived from an analytical model. Chapter V presents results for a special maneuver case and compares the results of the simulation to analytically-obtained results. Chapter VI describes the more complex maneuver model and some results. Chapter VII presents further results from the simulations which are used to draw conclusions of the usefulness and effectiveness of MAS systems and their limitations. Chapter VIII presents conclusions and suggestions for further research.

II. NEIGHBOURING DIAGONAL LOCAL AVOIDANCE MODEL (NDLAM)

A. INTRODUCTION

This chapter describes the discrete (“tiled”) version of the Minefield Objects Avoidance Maneuver (MOAM) model as mentioned by Kim (Kim 2002), and its simulation. This model was generalized by the Neighboring Diagonal Local Avoidance Model (NDLAM) as presented in Gaver *et al* 2003. The corresponding analytical model is used to partially validate the simulation. The analytical model is included in Appendix 1 for completeness so as to illustrate the algorithms followed by the discrete model.

B. DISCRETE MINEFIELD MODEL

The next few paragraphs will explain the differences between the simulation model of this thesis and that of Kim (2002). It will describe the model of the discrete minefield, how the mines and NOMBOs are distributed in this minefield, and how the detection of the mines, NOMBOs, and false alarms is implemented. It will also describe the general methodology of the simulation used.

1. Comparison between Previous Simulation and Current Approach

In his thesis, Kim (Kim 2002) used a spatial Poisson process to represent the positions of mines, NOMBOs and false alarms. For the spatial Poisson process simulation whether or not a ship encounters a mine, NOMBO or false alarm is simulated by drawing exponential random numbers to represent distances and comparing them to the length of the minefield. If the distance drawn is smaller than the length, then the ship is deemed to encounter a mine or NOMBO or false alarm, whichever the case may be. The avoidance maneuvers are simulated in a similar fashion if the vessel survives the encounter with a mine and is able to take avoidance actions. The ship does not actually “move”, and the randomness of the simulation is mainly manifested with the probability distribution of the distances drawn and whether or not an object is detected or a false alarm is generated. This is an event-based discrete event simulation in that the simulation change in state is triggered by the time (or distance) to next event, whether it is an encounter with a mine, NOMBO or false alarm.

In this thesis, a step-based discrete event simulation approach is taken The focus is on implementing more complex maneuvers that would better reflect realistic scenarios. As a check of the complex maneuvers, it was deemed necessary to obtain a visual representation of the minefield and the avoidance actions taken to verify the correctness.

2. Justification for Step-based Simulation Approach

The minefield is represented by a grid. In a Combat Information Center on a typical Mine Counter Measures Vessel (MCMV), the minefield area is usually represented as a square or grid on the screen. Synthetic lines are superimposed onto the area to show the path or track of the ship, usually lines of equal spacing to demarcate optimal paths (calculated by the Mine Information System) for minehunting or minesweeping to clear the area to the desired mine clearance confidence levels. The vessel will typically travel along one of these lines to conduct minehunting, and when an object is detected by the sonar system, the location is marked on the map with some symbol. If the mission of the vessel is to detect all the mines in the area for subsequent mine clearance, then the ship will go around the object, keeping some safe distance away from the object and proceed on its mission. This is similar in representation to the current tactic of object avoidance.

Without going into specifics of exact navigation, a grid model is deemed to represent such a minefield sufficiently well, as the vessel would typically go left, left-forward, right, right-forward, or astern when taking avoidance maneuvers. With spaces tiled with squares for simulation, a ship maneuver is simplified in that a platform "moves" in every time step of the simulation, may not detect the objects in an adjacent square sometimes in front of it, and would need to make "conscious" decisions on where to turn to if objects are detected in a square . This is can be easily represented on the screen. The randomness in the simulation is manifested on the minefield pattern drawn

(whether or not there is an object in a square), the probability of the ship detecting a mine or NOMBO, or the sensor giving a false alarm, and the resultant avoidance maneuvers of the ship.

C. STRUCTURE OF THE MODEL

1. Set-up of Minefield

In the model, the minefield has fixed length L and width W and therefore has a finite area. It is represented by tiling disjoint squares whose size is determined by the typical detection range of the sonar, assumed to be greater than the actuation mine radius of about 100m. For example, if a ship is trying to transit a narrow channel with a typical length of 4000m and a width of 3000m, and the detection range of the sonar is 200m, the minefield would consist of a grid of 20 rows (L) by 15 columns(W). Each square in this grid either contains a mine, a NOMBO or is empty; these events are mutually exclusive, i.e., a square cannot contain both a mine and NOMBO. Each square contains at most one object. See Figure 4 for the pictorial representation of a discrete minefield. In the example below, a 5 squares by 5 squares grid is used to represent a 1000m by 1000m minefield.

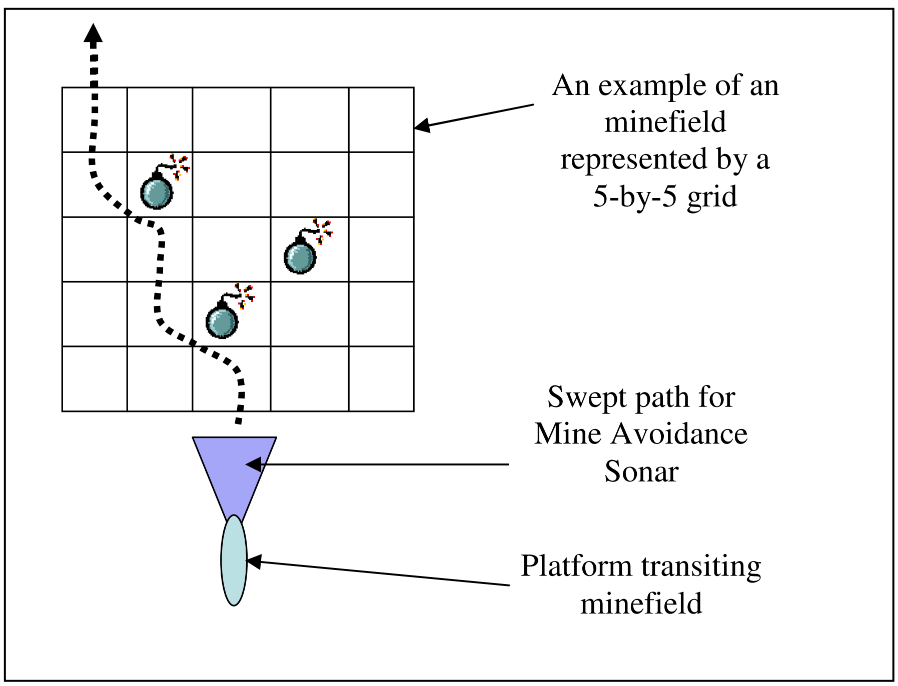

Figure 4. Pictorial Representation of a Discrete Minefield.

2. Dispersion of Mines and Objects in Minefield

Next, the mines and NOMBOs are sown into the minefield in a manner approximating a spatial Poisson Process, with small rate; the total number of objects (both mines and NOMBOs) must be less than the number of squares. First, with specified rates of mine density and NOMBO density for the whole area, random numbers drawn from a Poisson distribution will be used to represent the actual number of mines and NOMBOs in this minefield. With the numbers of the objects obtained for each type, each object will have a (row, column) coordinate that is derived from randomly drawn numbers from the specified rows and columns. If the square corresponding to the coordinate is empty, then the object would be "planted" there, and the attributes of the square changed to reflect that there is now an object in the square. Mines will be planted first, followed by NOMBOs. If a square has been occupied already, then a new coordinate is derived. This object-sowing process will repeat until all the mines and NOMBOs are planted. The pseudo code is shown below to illustrate this process. Figure 5 shows the parameters used for the pseudo code.

λ_m – rate of mines per unit area (in square nautical miles) – lambdaMines
λ_n - rate of nombos per unit area (in square nautical miles) – lambdaNombos
L – length of minefield in terms of rows
W- width of minefield in terms of columns
factor – number of times total area is larger than unit square mile

Repeat the following steps {

numberOfMines, numberOfNombos, totalNumberObjects = 0

numberOfMines : Draw E[number of mines] from Poisson process using a mean rate of (λ_m x factor)
numberOfNombos:Draw E[number of nombos] from Poisson process using mean rate of (λ_n x factor)

totalNumberObjects = numberOfMines + numberOfNombos

}
Until (totalNumberObjects < total number of squares in minefield)

:

Figure 5. Parameters For Pseudo Code On Sowing The Minefield With Mines And Nombos using Poisson distribution

Alternatively, objects can be planted first, and their identity i.e. mine or nombo decided when the object is detected by the vessel. However, this is not implemented here. Figure 6 shows how the mine sowing process is conducted using pseudo code to illustrate the steps taken in the simulation.

```
objectCount, mineCounter, nomboCounter = 0


Do {

    ratio = numberOfMines / (numberOfMines+numberOfNombos)


    Draw random number  U(0,L) for row coordinate y
    Draw random number U(0,W) for column coordinate x

    If (no object present in square[x,y])
        Plant object in square[x,y]
        Increment object counter objectCount by one

        Draw random number U(0,1)
        If (number < ratio)
                If (numberOfMines>0)
                        Set object in square [x,y] as mine
                        Increment counter mineCounter by one
                        Decrement numberOfMines by one
        Else
                If (numberOfNombos>0)
                        Set object in square [x,y] as Nombo
                        Increment counter nomboCounter by one
                        Decrement numberOfNombos by one

     ObjectCounter = mineCounter + nomboCounter

 }
```

Figure 6. Pseudo-code for dispersal of mines and NOMBOs into the discrete minefield

The next chapter will describe in more details the algorithms used in modeling the detection of mines, NOMBOs and false alarms and how they are checked, and also how the avoidance actions are taken by determining which position for the ship to go to next.

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III. SIMULATION MODEL FOR NEIGHBOURING DIAGONAL LOCAL AVOIDANCE MODEL (NDLAM)

A. GENERAL IMPLEMENTATION

The simulation is coded in Java using an object-oriented approach to generate the minefield, determine the path of a transiting vessel and generate objects other than mines. The simulation first implements the Neighboring Diagonal Local Avoidance Maneuvers (NDLAM) model presented in Gaver et. al. (2003), which was the discrete version of the Minefield Objects Avoidance Maneuvers (MOAM) model in Kim's thesis (Kim 2002). The NDLAM model considers a minefield that is infinite in the x-direction. When the ship returns to the beginning of the field because its passage has been blocked, it chooses a new starting position and attempts to cross the field without intersecting any of its previous paths.

A for-next loop is used to implement the simulation for the required number of replications, in this case, 10000 times in order to have small standard errors. At the start of an iteration, a minefield pattern is drawn. The minefield contains L rows and W columns. The first row of the mine field will be called row 1. The vessel will be situated in the middle square in row 0 of the minefield and makes its attempt to cross the minefield in accordance to the procedures described in Section A. When the vessel needs to backtrack (return to row 0), the vessel is relocated back to where it started, and a new minefield pattern is redrawn to mimic the new starting position of the ship in the NDLAM model.

```
Repeat for each Pd used, in increments of 0.1 from 0.1 to 1.0 the following steps {
        Reset random number stream to the same pseudo random number seed value.
        Next, repeat for desired number of iterations the following {

                Initialize vessel starting position
                Get a new minefield pattern
                Do {

                        Check the next square ship is heading to for mines, nombos or
                                false alarms
                        If (no detection)
                                Move ship to new square
                                If there is a missed mine
                                   Attempt fails
                                    Mine detonates, and the ship is destroyed
                        Else
                                Determine next position to go to
                                   If need to backtrack, redraw minefield

                While (ship has not crossed finish line and no detonations occur)
        }
}
```

Figure 7. Pseudo Code for main simulation process

This will repeat until the vessel crosses the minefield successfully or gets blown up by a mine due to a missed detection. This whole process will be repeated for a number of replications specified e.g. 10000 times in order to obtain small standard errors. Statistics are then collected on the number of mission successes (when an transit attempt is successful, regardless of the number of backtracks), the number of backtracks, the furthest distance the ship can reach before it backtracks, for both successful and failed outcomes. The simulation is repeated for different probabilities of detection, between 0.1 to 1.0. The pseudo code is shown in Figure 7. The random number stream is correlated for each pd used to better study the effects of pd and maneuvers affecting the mission success. Correlation is achieved by starting the random number streams with the same pseudo random number seed for different pds.

B. SIMULATION OF DETECTION

1. Simulation on Effectiveness of the Sonar Sensor

The effectiveness of a sonar sensor depends on many factors; for example the capability of its signal processor; the transmission strength of the sonar signal and the resolution of the receiver; the environmental conditions; the presence of marine life; the type of bottom textures; whether it is undulating, rocky; the composition of the bottom, whether it is sandy, muddy or hard; the number of minelike objects underwater, sea state, etc. The sensor cannot be used to classify the detected object as either a mine or a non-mine mine-like object. The effectiveness of the sensor as a function of the number of objects, mines and NOMBOS, and false alarms is the subject of this study; the other factors are not represented. . The sensor is characterized by its probability of detection, pd, and its false alarm rate A low probability of detection (pd) implies that the sensor is not sensitive and effective in detection of objects and so will not detect some objects. A higher pd implies that the capability of the sensor is better in object detection per scan or sweep of a square, and more objects will be detected. Referring to the analogy of the blind person, if the person were heavily distracted and tired, he or she would probably not sweep the stick around as rigorously, and would miss more objects due to carelessness. When he is more alert or vigilant, then the chance of the stick missing something would be lower.

2. Simulation on Detection of Mines and Nombos

Before the vessel moves into a new square, it will check to see if there is any minelike object ahead. A random number U(0,1) is drawn and compared against the specified Probability of Detection. If the random number is smaller than pd, and there is an object in the square, the object is detected and marked. The ship will then take avoidance maneuvers and not enter the square. Figure 8 shows the pseudo code for the simulated checking of mines and NOMBOs.

```
If the  ship is backtracking, there is no need to check the next square.

Check if there is an object in the next square to go based on heading direction

    If next square was not previously searched,
        Tag the square as searched
        Increment squares searched counter of ship by 1

    If there is an object present, simulate detection
        Draw a random number
        Check random number against prevalent pd used
        If random number is less than pd,
           Object is detected and check identity of object
                Detection is true;
                If it is a mine, tag object detected as mine
                If it is a nombo, tag object detected as nombo
                Block that direction
        Else
           Object is not detected
           Detection is false
```

Figure 8. Pseudo Code for checking of NOMBOs and mines

3. Simulation of False Alarms

The representation of false alarms differs somewhat from Kim, 2002. False alarms are assumed to be spurious contacts that are caused by noise spikes or natural phenomena that cause a detected contact to exhibit characteristics similar to that of NOMBOs and mines. In signal processing, the probability of detection can be increased by increasing the sensitivity of the sensors and lowering the threshold of the minimum detectable signal. This would however increase the chance of a noise spike being large enough to confuse the system into indicating that a contact has been detected, when there is nothing actually there. Thus the probability of a false alarm depends on the probability of detection. However, false alarms can also occur due to the presence of fish or the type and composition of the bottom which may give rise to echoes detected by the sonar and resemble mines and NOMBOs. Nevertheless, false alarms are transient in nature regardless of the source, and can be filtered away by the system if the sensor is able to classify the contact within a stipulated amount of time. We call the time from when the sensor starts to scan a single region until the operator determines there is or is not a mine-like object, the classification time.

If there is no object in the square, then a random number U(0,1) will be drawn to simulate the occurrence of false alarms. If this number is smaller than the probability of false alarm at the prevalent level of pd used, the false alarm is deemed to be have occurred. However, whether the false alarm would be detected by the system is dependent on how long the false alarm would last. If the occurrence of the false alarm is shorter than the typical classification time of the sonar system, the false alarm will not be detected. If the duration is longer than the classification time allowed, the false alarm is deemed to be an "object", and the ship will need to take avoidance actions to avoid this "object". A ROC curve using a standard detection index of 1.0 is assumed (Kim, 2002); the ROC curve relates the probability of detection (sensitivity of the sensor) to the false alarm rate where the probability of false alarms is dependent on the sensitivity of the sensor. A higher probability of detection would entail higher number of false alarms, and vice versa. The pseudo code is listed in Figure 9.

```
(Continued from above pseudo code)

        If there is no object present, simulate false alarms
                Check for detection of false alarms
                Draw a random number and compare against the
                        Probability of false alarm
                If number is less than the probability,
                        False alarm has occurred
                        Simulate the duration of the false alarm using
                                a Weibull distribution (see Figure 10)
                        If duration of false alarm is less than the system classification time,
                                false alarm is filtered and not detected
                                Detection = false;
                        Else
                                System is mislead to indicate false alarm as contact
                                Detection = true;
```

Figure 9. Pseudo Code for simulation of false alarms

The pseudo code for generating the duration of the false alarm is shown in Figure 10. The duration of false alarms is considered stochastic in nature, and the mean time duration is obtained from a Weibull distribution that is commonly used in reliability studies. Scale parameter α and shape parameter β, allow great flexibility in modeling the duration of a false alarm. The pseudo code is shown in Figure 10. This feature is included in the simulation model, but not exercised.

```
(Continued from above pseudo code Figure 9 on Weibull distribution)

    If false alarm has occurred, simulate duration of false alarms
        Draw a random number from Weibull Distribution using the formula
```

$$\text{Weibull Random Number} = \frac{1}{\alpha}(-\log U)^{\frac{1}{\beta}},$$

where U is random number ~ U(0,1),

α is the scale parameter and β is the shape parameter

```
        If number is more than or equal to the system classification time,
            false alarm is filtered and not detected
            Detection = false;
        Else
            System is mislead to indicate false alarm as contact
            Detection = true;
```

Figure 10. Pseudo Code for Generating Duration of False Alarms Using Weibull Distribution

C. SIMULATION OF MANEUVER

1. Description of Object Avoidance Maneuver

To carry out the avoidance maneuver, the ship needs to keep track of its position in the minefield, the direction in which the ship is looking in, the next square it needs to look at based on the current direction, and the information gathered from previous observations. Suppose the ship is in square with coordinates (x,y). The directions (squares) that a ship can look forward are (x+1,y), left-ahead (x+1,y-1), and right-ahead (x+1,y+1). The pseudo code is listed in Figure 11.

Determining next position

Get current position of ship in minefield; say (x,y)
Get information from previous determination of the presence or absence of objects in the squares ahead of ship.
A square is blocked if an object is detected or a false alarm occurs for longer than the classification time
If the square (x,y+1) is not blocked, the ship moves into the square and next square to check is (x,y+2) if the ship survives the move
If square (x,y+1) is blocked, set the next square to check is (x-1,y+1) . Etc…
If ahead and left-ahead blocked, set the next square to check as right-ahead

Figure 11. Pseudo Code for Object Avoidance Maneuver

If all three squares ahead (in the y-direction) of the ship are blocked, the minefield will be redrawn and the ship prepositioned at the starting position in row 0 to attempt once again to cross the field. This model is similar to having a ship go back to the starting point and move a number squares in the x-direction; if the new path of the ship never intersects the paths of previous attempts faced by the ship at that location the minefield pattern is probabilistically an independent identically distributed minefield. By redrawing the field, the need to devise special algorithms to handle boundary cases is eliminated. In the next chapter, we show the results obtained by the analytical model and compare them to the results from the discrete simulation.

D. RANDOM NUMBER STREAMS

For each square being checked for objects, random numbers are generated to compare against the prevalent pd used so as to simulate its detection. As pd increases from 0 to 1.0, holding the random number stream constant would allow us to better capture the effects of pd on detection of objects and influencing the avoidance tactics used. Therefore, the random number seed is reset to the same value in Figure 7 at the start of each change in pd. The resulting positively correlated random number streams will allow the variance between each change of pd to be reduced so as to better measure the variance due to the other factors. The random number streams will be held constant for the NDLAM model and the Special Case model of Chapter V.

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IV. COMPARISON OF THE SIMULATION RESULTS TO THOSE OF AN ANALYTICAL MODEL

A. INTRODUCTION

This chapter will compare the results of a simulation to those of an analytical model. A numerical example from the analytical NDLAM model (Gaver *et al* 2003) is used to evaluate the simulation results obtained, where the mathematically computed probability of safe minefield transit is compared to the proportion of simulation replications which result in successful minefield transits.

B. PARAMETERS USED

For a detailed description of the analytical model, please see Appendix 1. The rate of occurrence of mines $\boldsymbol{\lambda_m}$ and NOMBOs $\boldsymbol{\lambda_n}$ per square nautical mile used in this numerical example is 5 per square nm and 10 per square nm respectively, with a total object rate of 15 objects per square nautical mile. The minefield length and width is 5 nm each, comprising of 2500 squares (50 squares lengthwise and 50 widthwise). This area is 25 square nautical miles, a factor of 25 times greater than the unit square nautical mile which the rates of mines and NOMBOs are based upon. To obtain the total number of mines for the whole area, the rate of 5 mines per square nautical mile is multiplied by the factor of 25 to get a rate of 5x25 = 125. With this rate of 125 mines per 25 square nautical miles, the number of mines in the area has a Poisson distribution with mean 125. The total number of NOMBOs is also obtained in this fashion, using the rate of 10 NOMBOs per square nautical mile to obtain a total rate of 10*25 = 250 NOMBOs per 25 square nm. The number of NOMBOs has a Poisson distribution having mean 250. The total number of objects obtained is thus the sum of the drawn number of mines and the drawn number of NOMBOs.

Each square is 200 yards long and wide, reflecting the detection range of the sonar system. The conditional probability of detection of an object given an object is in a square, δ, is varied from 0.1 to 1.0. $\boldsymbol{\beta}$**,** the probability of an object in a square, is simply the mean total number of objects (mean number of NOMBOs +mean number of mines) divided over the number of squares. $\mathbf{P_m}$, the probability of the object in a square being a mine if object is detected is calculated as the ratio of the mean of the number of mines over the combined means of mines and NOMBOs. In this case, $\mathbf{P_m}$ = 125/(375) = 0.33. $\boldsymbol{\gamma}$ is the probability that there is no detection in the next square, either because there is no object or because the object is not detected. $\boldsymbol{\alpha}$ is the probability of successfully moving 1 square ahead in the y-th direction, and $\boldsymbol{\varphi}$ is the probability that all 3 squares ahead have objects in them that are detected. κ_M is the probability the ship will be killed when entering a square. The probability the vessel V crosses the minefield of length in the y-direction L, without being blown up is **P(D>L),** which is also the MOP we are using to evaluate the probability of mission success, where D is the number of squares safely transited in the y-direction.

1. Results

The table below shows an output from the discrete NDLAM mathematical model:

δ	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
β	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150	0.150
p_M	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333	0.333
γ	0.950	0.940	0.930	0.920	0.910	0.900	0.890	0.880	0.870	0.860	0.850
α	0.950	0.954	0.959	0.963	0.968	0.973	0.977	0.982	0.987	0.992	0.997
ϕ	0.000	0.000	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.002	0.003
Km	0.050	0.046	0.041	0.037	0.032	0.027	0.022	0.017	0.011	0.006	0.000
P(D>=L)	0.077	0.097	0.122	0.154	0.196	0.252	0.325	0.422	0.553	0.736	1.000

Table 1. Probability of Successful Minefield Transit from NDLAM mathematical model

The chart illustrating the results is shown in Figure 12. The results show that the probability of mission success appears to be increasing exponentially with an increase of probability of detection for the mine avoidance sonar system, holding other factors constant.

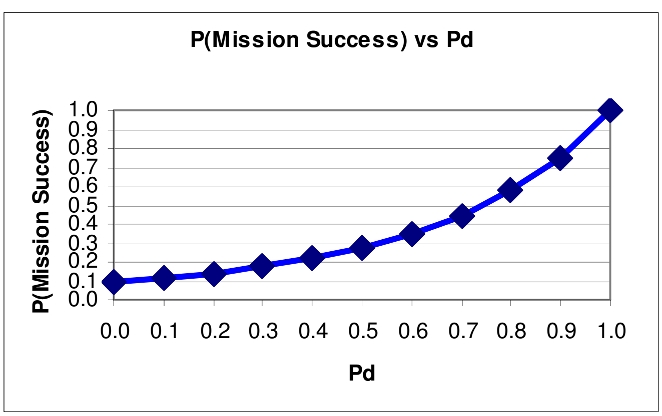

Figure 12. Chart showing output from the discrete NDLAM analytical model

Next, we look at the results of the discrete simulation for the NDLAM model to validate the results of the analytical model, and vice versa.

C. RESULTS FROM NDLAM SIMULATION

The simulation was conducted with the same parameters described above for 10000 replications times for each pd from 0.1 to 1.0. For each replication different random numbers are drawn from a Poisson distribution with the same rate of mines and NOMBOs to represent mines and NOMBOs. The mean number of mines, NOMBOs and objects for the simulation is shown in Table 2. It must be noted that the random number stream is correlated for each pd used, i.e. the random number stream is reset to the same value for the pseudo random number seed, for every change of pd so that the outcome of each transit is determined by the prevalent pd and the avoidance tactics rather than due to the variation of the random numbers. This would result in smaller standard errors.

Probability of sensor detection	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Mean no of objects in area	375.1	374.6	374.7	374.9	374.8	375.2	374.9	374.8	375.0	375.1
Std Error no of objects	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
Avg no of mines in area	125.2	124.8	124.9	124.9	124.9	125.1	125.1	124.9	125.1	125.0
Std Error no of mines	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
Avg no of NOMBOs in area	249.9	249.8	249.8	250.0	249.9	250.2	249.8	249.9	250.0	250.1
Std Error no of NOMBOs	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

Table 2. Distribution of mines and NOMBOs for NDLAM model

The results are shown in Table 3. It is observed that the probability of mission success increases with the probability of detection, similar to the analytical results. The standard errors achieved are small i.e. smaller than 0.011, which shows a tight fit of the simulation data around the mean.

Probability of Detection	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P(Mission Success)	0.069	0.097	0.118	0.157	0.196	0.252	0.321	0.423	0.553	0.727	1.000
Std Error of P(Success)	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.004	0.000
+95% CI	0.074	0.103	0.124	0.164	0.204	0.261	0.330	0.433	0.562	0.736	1.000
-95% CI	0.064	0.091	0.112	0.150	0.189	0.243	0.312	0.414	0.543	0.718	1.000

Table 3. Results for Simulation of NDLAM Model

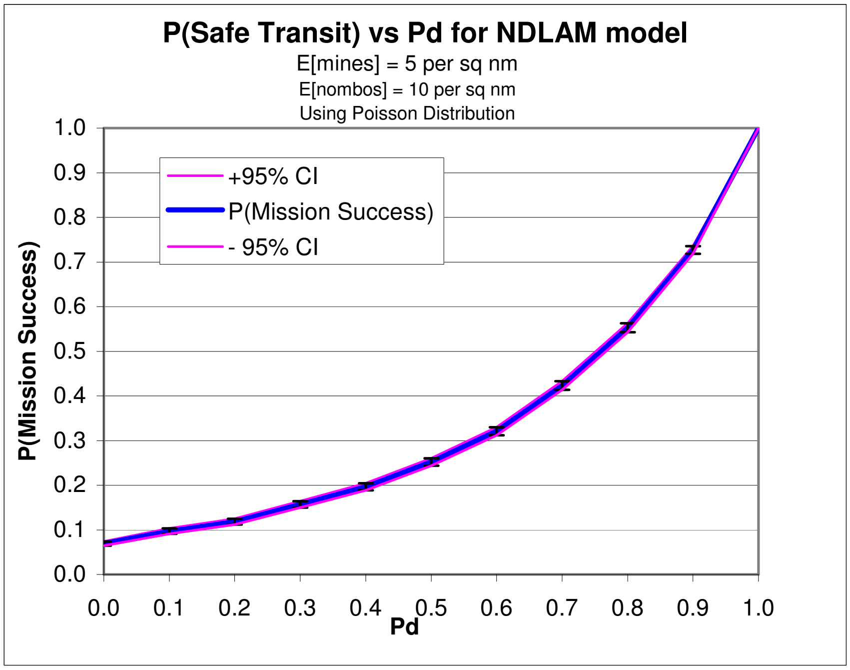

Figure 13. Probability of Mission Success vs. Pd for NDLAM Simulation

The output from the actual Java program is shown in Appendix 2, where the critical portions are extracted and written into an EXCEL file for data analysis and for graphically displaying the results.

D. COMPARISON OF RESULTS

1. Probability of Mission Success

For comparison, Figure 14 displays the results from the analytical study and the simulations on the same graph. As can be seen from the graph, the two curves are close to each other, with the analytical curve contained within the 95% confidence interval of the simulation curve.

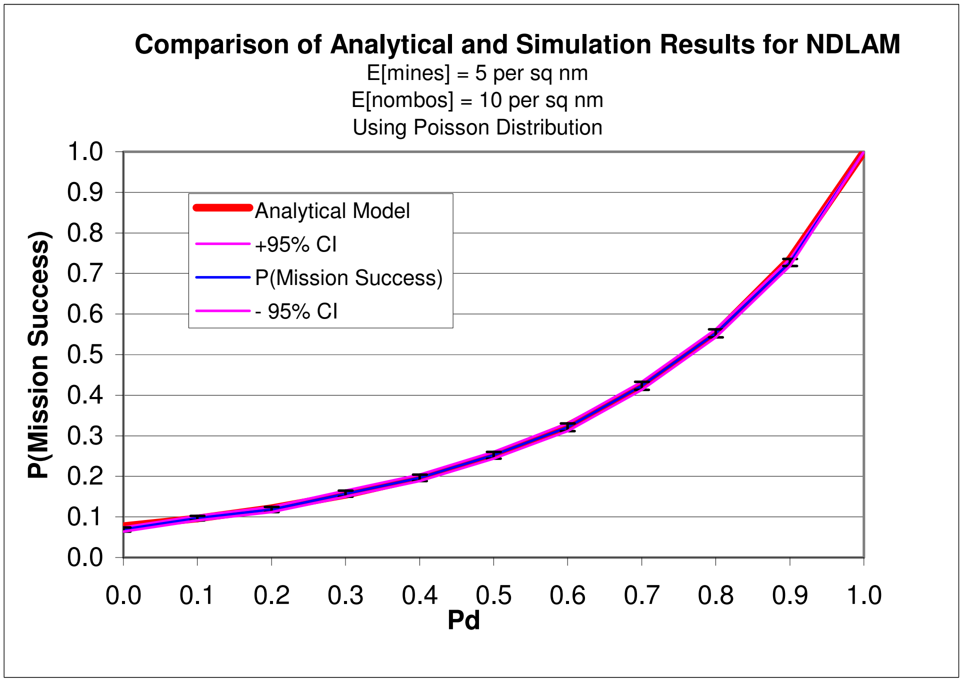

Figure 14. Graph of results for the analytical model and discrete NDLAM model

The detailed results are captured in Table 4, where the Probability of Mission Success, its standard error, the upper and lower confidence intervals are shown along the results from the analytical calculations.

Probability of Detection	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P(Mission Success)	0.069	0.097	0.118	0.157	0.196	0.252	0.321	0.423	0.553	0.727	1.000
Std Error of P(Success)	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.004	0.000
+95% CI	0.074	0.103	0.124	0.164	0.204	0.261	0.330	0.433	0.562	0.736	1.000
-95% CI	0.064	0.091	0.112	0.150	0.189	0.243	0.312	0.414	0.543	0.718	1.000
Analytical P(Mission Success)	0.077	0.097	0.122	0.154	0.196	0.252	0.325	0.422	0.553	0.736	1.000

Table 4. Comparison of Simulation and Analytical Results

2. Other Simulation Results

a. Average Number Of Backtracks

To examine the effectiveness of this maneuver, the average number of backtracks i.e. restarting from the initial position with a new minefield whenever the path ahead is thoroughly blocked, and the maximum row (signifying distance) reached were also measured. The empirical distribution of the number of backtracks can also be obtained but is not done here. The simulation results are shown in Table 5 next:

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Average Number of backtracks	0	0.000	0.000	0.001	0.002	0.004	0.011	0.021	0.039	0.070	0.109
Std Error Number ofBacktracks	0	0.000	0.000	0.000	0.000	0.001	0.001	0.001	0.002	0.003	0.003
+95% CI	0	0.000	0.000	0.001	0.003	0.006	0.013	0.024	0.042	0.075	0.116
-95% CI	0	0.000	0.000	0.000	0.001	0.003	0.009	0.018	0.035	0.065	0.103

Table 5. Simulation Results for Average Number of Backtracks for NDLAM

The graph for the simulation results of average number of backtracks i.e. restarting back from the initial position when the front three squares are blocked is shown in Figure 15. The graph shows that the average number of backtracks increases exponentially with the increase in probability of detection. For example, when pd = 0.1, the average number of backtracks is 0. This is logical because the sensor cannot detect anything, so the ship will just bash through the minefield in a straight path and will not take avoidance actions because it is not able to detect anything. The chances of the vessel executing a backtrack is proportionately lower since the chances of finding all 3 squares ahead being blocked is low. Therefore, the probability of safe minefield transit is understandably lower because of the increased chance of being blown up due to the lack of avoidance actions being taken. On the other hand when pd = 1.0, the ship will often take avoidance actions since it can detect everything. However, even although there is an increase in the average number of backtracks i.e. to 0.11, the number is low because this specific mine and NOMBO density (5 mines and 10 NOMBOs per square nm) does not necessitate the execution of more backtracks.

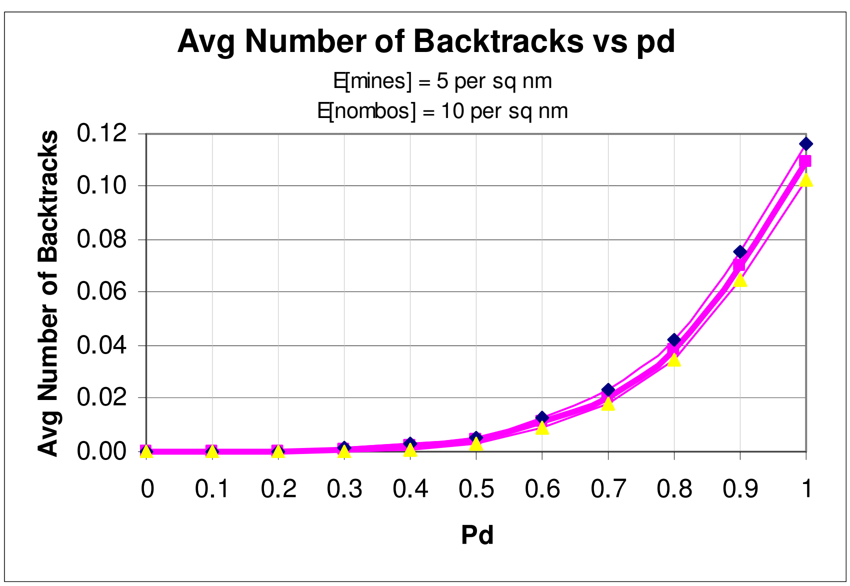

Figure 15. Average Number of Backtracks

b. Average Maximum Proportion of Minefield Reached Given Mission Failure

Table 6 shows the results for the average maximum distance traveled in the y-direction through minefield (represented by rows) before mission failure for those replications in which the ship encountered an undetected mine. This gauges how far a ship can go before it is destroyed. To compute how far the vessel will travel in the y-direction, the simulation program keeps track of the maximum distance traveled in terms of maximum rows reached for each replication that ends in mission failure. For example, a vessel may or may not backtrack in 1 replication. If the row reached is 20, and it backtracks, and the subsequent row reached is 15 rows before it detonates, then the maximum distance reached for that replication is 20. This maximum distance is stored and then averaged over the number of replications for each pd that ends in failure. From the average maximum number of rows reached, the proportion in the y-direction of the minefield covered is then computed. For example, if the average maximum row reached is 17.63, when pd = 0.1, then the corresponding average maximum proportion of minefield reached is 17.63/50, assuming there are 50 rows. The results are displayed in Figure 16.

Probability of Detection	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Average Max Distance Travelled	16.829	17.630	18.347	19.185	19.753	21.083	21.834	22.847	24.098	25.953	0.000
Std Error Distance Travelled	0.130	0.134	0.138	0.146	0.150	0.159	0.171	0.185	0.215	0.276	0.000
+95% CI	17.083	17.892	18.618	19.471	20.047	21.396	22.168	23.210	24.519	26.494	0.000
-95% CI	16.574	17.368	18.076	18.900	19.459	20.771	21.499	22.483	23.677	25.413	0.000
Avg Max Proportion Minefield reached	34%	35%	37%	38%	40%	42%	44%	46%	48%	52%	0%

Table 6. Table of results for Average Maximum Proportion of Minefield Reached Given Mission Failure for a minefield having 50 squares in the y-direction

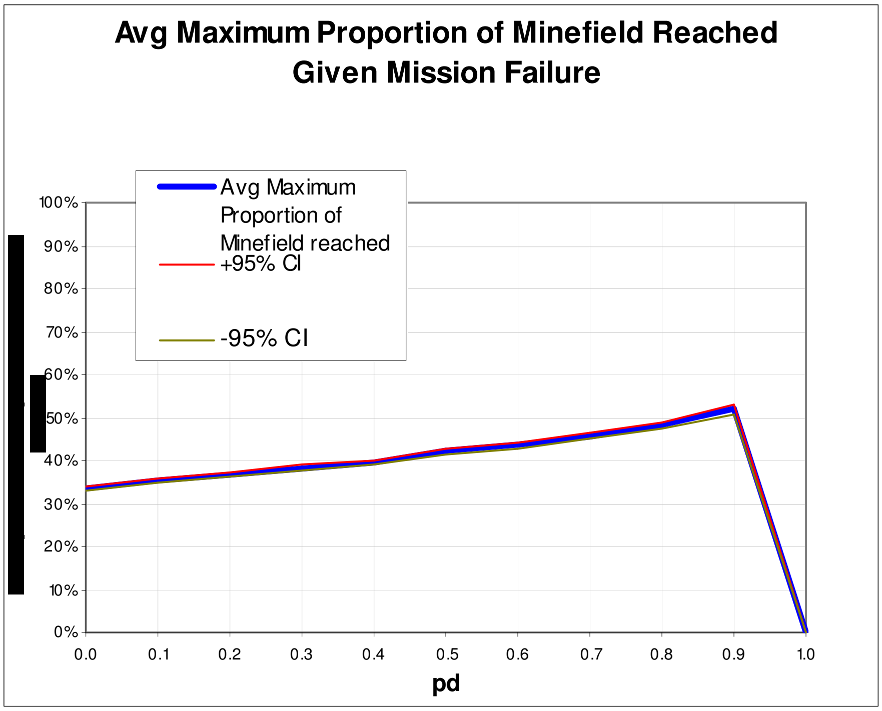

Figure 16. Average Maximum Proportion of Minefield reached before Mission Failure

In the example given, the proportion ranges from 34% when pd is 0, to 52% when pd is 0.9 . When pd = 1.0, the vessel will always succeed in transiting the minefield, so the average maximum proportion of minefield reached given mission failure is 0. These results illustrate that the MAS is effective in getting the ship further into the minefield before it encounters an undetected mine e.g., about 52% when pd=0.9. This is compared to the situation where there is no MAS or when pd=0, where only 34% can be reached

E. FURTHER INVESTIGATION ON THE EFFECT OF DISTRIBUTION OF MINES AND NOMBOS

The simulation model of the previous sections of this chapter assumes that the numbers of mines and NOMBOs have independent Poisson distributions. To investigate if the underlying assumption of the distribution would have any effect on the probability of safe minefield transit, an additional simulation was conducted assuming that the number of mines and NOMBOs followed a Binomial Distribution instead, but using similar parameters. Modifying Figure 5 from Chapter 2, the numbers of NOMBOs, mines and total objects were drawn from Binomial Distributions with the parameters and procedures described below. As before, common random number streams are used for each change in pd.

1. Parameters Used

Let the number N be the number of squares in the area, each square being 200 yards long and wide. The probability of a square containing an object would be the total rate of objects for the specified area (in squares), divided by the total area (in squares). For example, if a minefield is 5nm by 5nm, the total number of squares would be 50 x 50 = 2500 squares (1 nm = 2000yards = 10 squares long). This area is therefore 25 times the area of a square mile (1 square mile has 100 squares). The total rate of objects would then be the sum of the rate of mines per square mile + rate of NOMBOs per square mile, multiplied by the factor the specified area is larger than a square mile. Using the same rate of 5 mines per square nm and 10 NOMBOs per square nm, the total rate would be 15x 25 = 375 objects. The probability of a square containing an object would therefore be 375/2500 = 0.15. The conditional probability that the object, if detected is a mine is the ratio of the rate of mine per square nm, to the sum of the rate of mine per square nm and the rate of NOMBO per square nm, i.e. 5/(5+10) = 0.333. The number of objects, ***Obj*** was first drawn from a Binomial distribution with parameters, the number of squares N=2500, and p = 0.15. The number of mines ***m*** is drawn next, with parameters N= ***Obj*** and p = 5/(5+10) = 0.33. Note that there will only be 1 NOMBO or a mine in a square, and not both. The number of NOMBOs ***n*** is then equal to the difference between ***Obj*** and ***m***.

2. Pseudo Code For Number Generation From Binomial Distribution

The pseudo code is shown below in Figure 17:

λ_m – rate of mines per unit area (in square nautical miles) – e.g. 5
λ_n - rate of nombos per unit area (in square nautical miles) – e.g. 10
L – length of minefield in terms of rows e.g. 50
W- width of minefield in terms of columns e.g. 50
Area Scale factor – number of square nautical miles in the finite minefield e.g. 25

N – number of trials (number of squares or objects)
Obj – number of objects
m – number of mines
n – number of nombos

Obj : Draw E[number of objects] ~ Binomial (N=L*W, p= [(($\lambda_m + \lambda_n$)* area scale factor) / (L*W)])

m : Draw E[number of mines] ~ Binomial (N=Obj, p= $\lambda_m / (\lambda_m + \lambda_n)$)

n : ***Obj – m*** (i.e. number of NOMBOs is the difference between objects and mines

Figure 17. Parameters For Generation Of Numbers Of Nombos And Mines From Binomial Distribution

3. Results from Simulation

Table 7 shows the results obtained for the simulation(first table) using a Binomial distribution, comparing against those in the 2nd table (from Table 4) from a Poisson Distribution:

Probability of Detection	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P(Mission Success) (Binomial)	0.096	0.123	0.154	0.197	0.246	0.328	0.414	0.547	0.734	1.000
Std Error of P(Success)	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.004	0.000
+95% CI	0.102	0.130	0.161	0.205	0.254	0.337	0.423	0.557	0.743	1.000
-95% CI	0.090	0.117	0.147	0.189	0.237	0.319	0.404	0.537	0.726	1.000
Error amount (1.96x Std Error)	0.006	0.006	0.007	0.008	0.008	0.009	0.010	0.010	0.009	0.000
Analytical P(Mission Success)	0.097	0.122	0.154	0.196	0.252	0.325	0.422	0.553	0.736	1.000

Probability of Detection	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P(Mission Success) Poisson	0.102	0.125	0.160	0.206	0.255	0.330	0.419	0.544	0.737	1.000
Std Error of P(Success)	0.004	0.005	0.005	0.006	0.006	0.007	0.007	0.007	0.006	0.000
+95% CI	0.110	0.134	0.170	0.217	0.267	0.343	0.433	0.558	0.749	1.000
-95% CI	0.093	0.116	0.150	0.195	0.243	0.317	0.405	0.530	0.725	1.000
Error amount (1.96x Std Error)	0.008	0.009	0.010	0.011	0.012	0.013	0.014	0.014	0.012	0.000
Analytical P(Mission Success)	0.097	0.122	0.154	0.196	0.252	0.325	0.422	0.553	0.736	1.000

Table 7. Comparison of Simulation Results from Different Distributions Used

The other results for the average number of backtracks and average maximum rows reached before backtracking given the ship encounters an undetected mine are also very close. The difference between the two models is very small, and the simulation results from the Binomial Distribution are contained within the confidence intervals of the results from the Poisson Distribution.

F. DISCUSSION

The simulation output compares well with the analytical results for the one case examined using the probability of safe minefield transit as the measure of effectiveness (MOE), regardless whether the underlying distribution is assumed to be Poisson or Binomial. The analytical calculation results are within the 95% confidence interval of the simulation result with the same inputs. This suggests that the NDLAM discrete simulation is consistent with the analytical NDLAM model with these parameters.

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V. SPECIAL CASE FOR ENHANCED MANEUVER MODEL (EMM)

A. INTRODUCTION

This chapter will present a special case of the Enhanced Maneuver Model (EMM) with an analytical model (Jacobs, 2005) developed to bridge the differences between the NDLAM and EMM. The purpose of this model is to provide a basis for partially validating the forward checking portions of the EMM model on the probability of safe passage.

B. DESCRIPTION OF MODEL

In the NDLAM model, the ship checks the front 3 squares only. It backtracks to the start of the field if all 3 squares ahead are blocked. The EMM model checks for the same 3 squares as well. However, if all three front squares are blocked, the ship checks the left and right squares of the row the ship is in to attempt to go around the blocked square. In the simulation model of this chapter, the minefield is 3 squares wide. The ship will not go backwards. If the ship cannot go forwards because it has detected objects in all adjacent squares , the simulation replication is scored as a failure and ended. The corresponding analytical model results in a recursive procedure to compute the probability a ship safely transits this special minefield that is 3 squares wide and L squares deep for a maneuver with no backtracking. Appendix 3 gives the details of the calculations. The measure of performance, P(D>L) is the probability that the ship can cross the minefield OF length L (in rows) safely without being blown up or blocked There are basically two initial ship positions: the vessel can start from the middle or start from the edges. The results are not expected to differ much.

C. IMPLEMENTATION OF SIMULATION

To implement this model, the simulation code for the NDLAM model in Figure 7 was used as the base, with specific portions amended. The total number of objects has a binomial distribution and the total number of objects, mines and NOMBOs were generated as presented in Chapter IV Figure 17. For the main program, the simulation was conducted for each of the 3 initial positions (left, center and right), when the pd would be varied per position. For each pd, the vessel attempts to cross the minefield for the number of replications specified, in this case 10,000. The minefield width was also amended to 3 squares wide i.e. 600 yards width. For the length L, an arbitrary number of 20 rows was chosen for ease of comparison. See Figure 18 for the pseudo code. Common random number streams are used for each pd for reasons described in Chapter II.

```
1.    For each starting position, perform the following:

2.    Repeat for each Pd used, in increments of 0.1 from 0.1 to 1.0 the following steps

3.    Reset the random number streams to the same pseudo random seed value each time.

4.    Next, repeat for desired number of iterations the following
{

          Get vessel's starting position from array and place it there
          Do {

               Check the next square ship is heading to for mines, nombos or
                    false alarms
               If (no detection)
                    Move ship to new square
                    If there is a missed mine
                       Attempt fails
                        Mine detonates, and the ship is destroyed
               Else
                    Determine next position to go to

          }
          While (ship has not crossed finish line and no detonations occur)
          Record failure or success and other statistics
          Reset variables and transit history on minefield.
}
```

Figure 18. Pseudo Code for Main Simulation code for Special Case Implementation.

The maneuvering portion in "Determine the next position to go" was amended so that there is no backtracking, that is, the vessel will only check all the forward and side squares to find a way through. If all the ways are blocked, then the vessel will not move back one square where it came from to look for other paths, which is done in the EMM model. Instead, the attempt will be considered a failure if all squares ahead and to the sides are blocked. The replication is over. Please see Figure 19 for the pseudo code for determining the next position to go to.

Determining next position to go

Get Information
1. Get current position of ship in minefield; say (x,y)
2. Get information from previous determination of the presence or absence of objects in the squares ahead of ship.

Conditions for blocked square
3.. A square is blocked if an object is detected or a false alarm occurs for longer than the classification time

Checking available pathways for all 5 directions
4. If the square (x,y+1) is not blocked, the ship moves into the square and next square to check is (x,y+2) if the ship survives the move
5. If square (x,y+1) is blocked, the next square to check is (x-1,y+1) . Etc…
6. If the squares ahead and left-ahead blocked, the next square to check as right-ahead (x+1,y+1) and etc.
7. If the front 3 squares are blocked, check the left square i.e. (x-1,y). If the square is not blocked, the ship moves into the square. If it survives the move, the next square to check is (x-1,y-2), etc. since (x-1,y-1) was found blocked earlier on.
8. If the front 3 squares and the left square is blocked, check the right square (x+1,y) and perform similar checks as for the left square.

Handling of blockages
9. If all 5 directions are blocked i.e. (front, left-ahead, right-ahead, left, right blocked) Transit attempt has failed. Increment failure counter.
10. There is at least one way that is open. Return to main loop of main simulation to pursue the available direction

Figure 19. Pseudo Code for Avoidance Maneuver for Special Case Implementation.

D. RESULTS

1. Parameters Used

The expected mine and NOMBO density is 5 mines and 10 NOMBOs in the 3x20 field, respectively. Independent minefields are drawn using these parameters for each replication, using the procedures described in Figure 17 in the previous chapter. Let D be the additional distance the ship can travel in the vertical direction until it encounters an undetected mine or is completely blocked. Let X be the current position of the ship the number of squares. We want to estimate P(D>L), i.e. assuming that the minefield is L rows long, what is the probability the vessel can travel safely for at least L rows in the vertical direction. Since L =20 rows deep, the number of squares in the minefield= 3x20=60 squares. Therefore the probability there is an object in the square, β is 15/60 =

0.25. The conditional probability that the object, if detected, is a mine is 5/(5+10) = 0.33. δ is conditional probability the object is detected, given there is an object in the square, and is varied from 0.0 to 1.0. The probability that a ship can safely enter a square is $\gamma = 1 - \beta + \beta(1-\delta)(1-p_M)$. When the ship is within the minefield in square (M,z), i.e. in the middle square (x-direction), z squares into the field (y-direction). $P_M(D > y)$ is the probability the ship can travel at least y more squares vertically without encountering an undetected mine or being completely blocked. When the ship is in square (L,z), i.e. in the leftmost square, z squares into the field (y-direction). $P_E(D > y)$ is the probability the ship can travel at least y more squares vertically without encountering an undetected mine or being completely blocked given it is currently in an edge square, e.g. (L,z), and square (M,z) has not been looked into by the ship sensor. $P_{EB}(D > y)$ is the conditional probability the ship in a edge square (e.g. (L,z)) will travel at least y squares vertically (in the y-direction) without encountering an undetected mine or being completely blocked, given the ship has already found square (M,z) blocked.

2. Analytical Results

Results from the analytical model are shown in Table 8:

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Gamma	0.9167	0.9000	0.8833	0.8667	0.8500	0.8333	0.8167	0.8000	0.7833	0.7667	0.7500
Pm(D>20)	0.1755	0.2016	0.2321	0.2670	0.3064	0.3498	0.3965	0.4457	0.4958	0.5453	0.5920
Pe(D>20)	0.1755	0.2014	0.2317	0.2664	0.3053	0.3482	0.3944	0.4427	0.4918	0.5400	0.5852
Peb(D>20)	0.1755	0.2013	0.2312	0.2652	0.2652	0.2652	0.2652	0.4339	0.4796	0.5239	0.5648

Table 8. Analytical results from Special Model

We see from the plot in Figure 20 for the results of Table 8 that the probability of safe minefield transit is almost linearly increasing with increases in pd. This agrees with the trend that the probability of the vessel crossing the minefield safely should increase with larger pd as it can take more effective avoidance maneuvers, but there is a limit to how many avoidance actions can be taken, as the chance of getting fully blocked is high. Even with a pd of 1.0, the vessel is predicted to have about a 60% chance to cross the minefield safely,; due to the very narrow minefield this does not leave room for much evasive maneuvering.

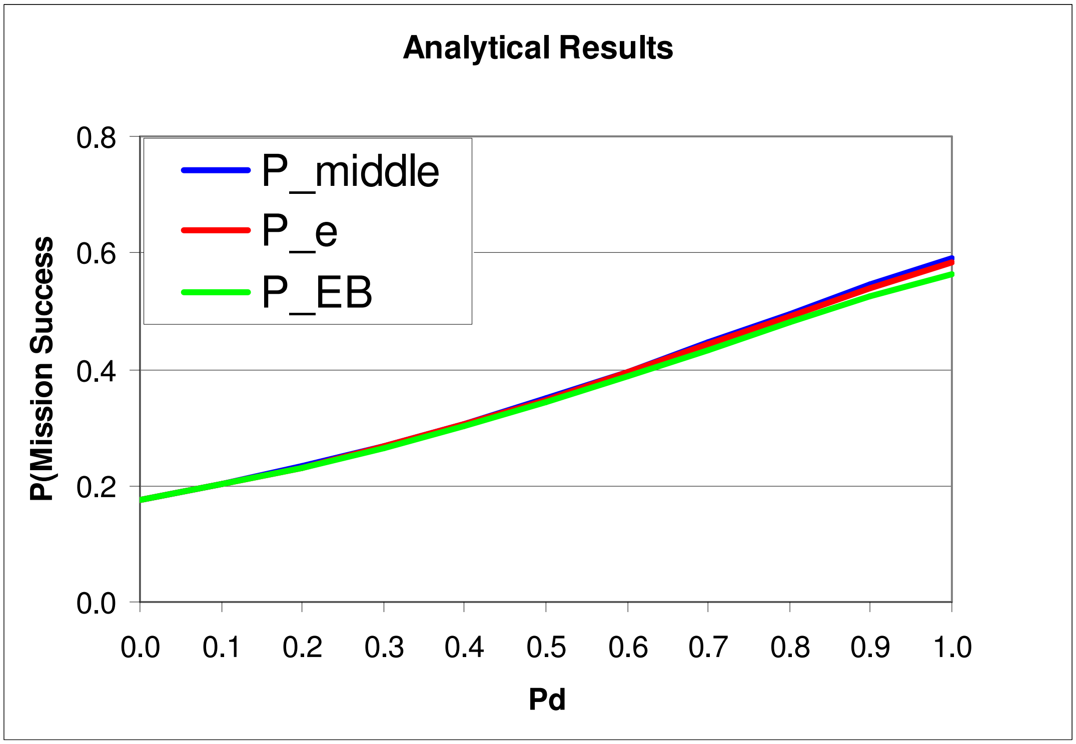

Figure 20. Plotted Results from the analytical model for Special case

On the other hand, if the ship is not equipped with any sensor or if the sensor is not functional, the vessel has about 17% probability of crossing the field safely due to the many possible minefield layouts of the 5 mines and 10 nombos. In this case, the ship proceeds straight through the field with no avoidance maneuvering and the transit is successful if no mines are encountered.

3. Simulation Results

The tabulated simulation results are shown in Table 9 and plotted in Figure 21 for the different starting positions for this minefield that is 20 rows and 3 columns wide.

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Leftmost	0.1737	0.2009	0.2296	0.2689	0.3059	0.3527	0.3978	0.4430	0.4973	0.5466	0.5870
Std Error	0.0027	0.0029	0.0030	0.0031	0.0033	0.0034	0.0035	0.0035	0.0035	0.0035	0.0035
+95% CI	0.1790	0.2065	0.2355	0.2751	0.3122	0.3593	0.4046	0.4499	0.5042	0.5535	0.5938
-95% CI	0.1684	0.1953	0.2237	0.2627	0.2995	0.3460	0.3910	0.4361	0.4904	0.5396	0.5802
Error amount	0.0053	0.0056	0.0059	0.0062	0.0064	0.0066	0.0068	0.0069	0.0069	0.0069	0.0068
Center	0.1704	0.20135	0.23695	0.26635	0.30175	0.35335	0.40415	0.4495	0.49395	0.5485	0.59365
Std Error	0.0027	0.0029	0.0030	0.0031	0.0033	0.0034	0.0035	0.0035	0.0035	0.0035	0.0035
+95% CI	0.1757	0.2069	0.2428	0.2725	0.3081	0.3600	0.4109	0.4564	0.5009	0.5554	0.6005
-95% CI	0.1651	0.1958	0.2311	0.2602	0.2954	0.3467	0.3974	0.4426	0.4870	0.5416	0.5868
Error amount	0.0053	0.0056	0.0059	0.0062	0.0064	0.0066	0.0068	0.0069	0.0069	0.0069	0.0068
Rightmost	0.17465	0.2051	0.2342	0.27045	0.30455	0.3496	0.3953	0.44625	0.49585	0.54475	0.5915
Std Error	0.0027	0.0029	0.0030	0.0031	0.0033	0.0034	0.0035	0.0035	0.0035	0.0035	0.0035
+95% CI	0.1799	0.2107	0.2401	0.2766	0.3109	0.3562	0.4021	0.4531	0.5028	0.5517	0.5983
-95% CI	0.1694	0.1995	0.2283	0.2643	0.2982	0.3430	0.3885	0.4394	0.4889	0.5378	0.5847
Error amount	0.0053	0.0056	0.0059	0.0062	0.0064	0.0066	0.0068	0.0069	0.0069	0.0069	0.0068

Table 9. Simulation Results for Special Case

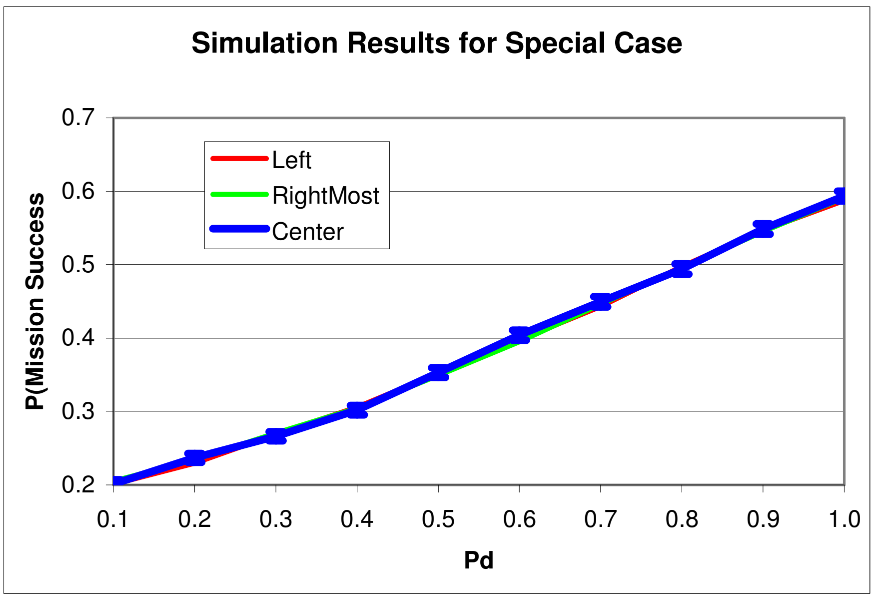

Figure 21. Chart of plotted simulation results for Special Case

We can see from the table and also the plots in Figure 21 that the plots of the 3 lines corresponding to the positions are all very close to one another, as the results from the left and right positions are contained within the 95% confidence intervals of the center position, indicated by the error bars. From the results there seems to be no significant differences in the starting positions affecting the outcome for these parameters of 5 mines and 10 NOMBOs per square nm. However, the analytical results show that there is a small difference for these parameters values.

E. COMPARISON

The results are plotted together in Figure 22. We see that they are almost identical, and all the curves are generally within the 95% confidence interval of the simulation results for the center position

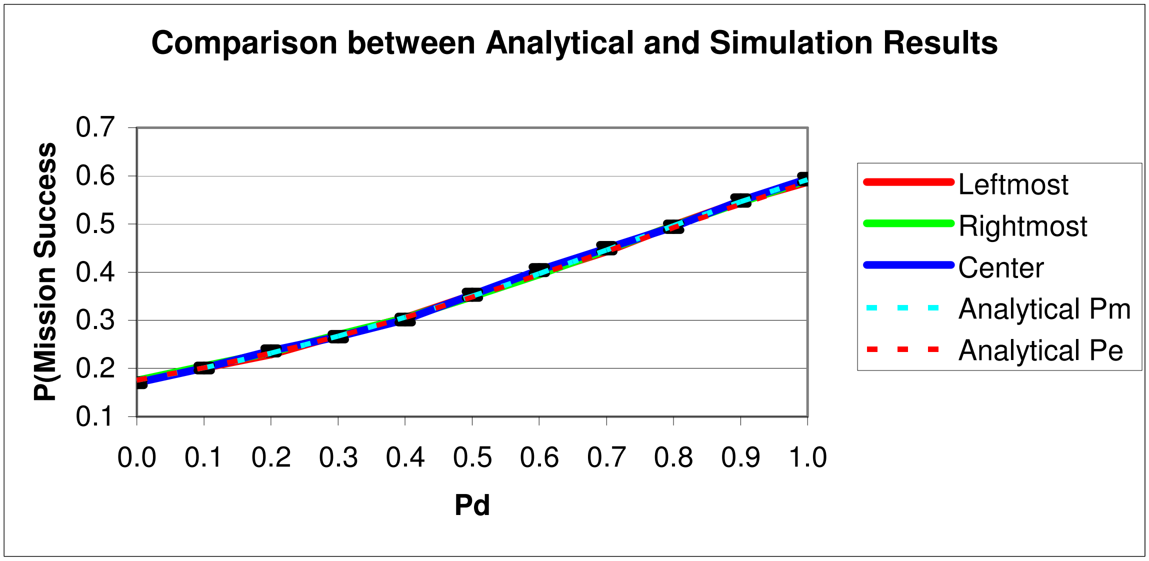

Figure 22. Chart of Comparison between Analytical and Simulation Results for Special Case

F. DISCUSSION

The results comparing the analytical model and the simulation are very close to each other, with small standard errors, and the analytical results are generally within a 95% confidence interval of the simulation results. This indicates that the simulation is consistent with the special case analytical model for the parameters chosen.

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VI. ENHANCED MANEUVER MODEL (EMM)

A. INTRODUCTION

This chapter describes the more realistic and new, Enhanced Maneuver Model and how it is used for the simulation of mine avoidance. The purpose of developing this new model is to improve the representation of the maneuvering tactics used in NDLAM to yield more realistic results that could better evaluate the effect of the density of NOMBOs and mines on the effectiveness of mine avoidance sonars. We describe the general simulation method and then explain the enhanced maneuver.

B. DESCRIPTION OF MODEL

1. Differences between EMM and NDLAM

a. Handling of Blockages

The Enhanced Maneuver Model (EMM) is based upon the NDLAM model, but will simulate more realistic maneuvers similar to those a real ship could execute when faced with a similar scenario. In the previous models (the Simple Minefield Transit Model (Kim, 2002) and NDLAM), the avoidance tactics executed were unrealistic since when the ship is blocked, the ship backtracks all the way back to the beginning of the field. A more realistic approach would be to check for objects in adjacent squares in the x-direction to attempt to go around the 3 blocked squares, before retracing the steps.

b. Backtracking

Also, instead of backtracking all the way back to the beginning of the field a ship could move a certain distance backwards and try checking the adjacent squares first for a path leading ahead by going around the 3 blocked squares. In this manner the ship would minimize the distance traveled and continue moving forward at every opportunity to transit the minefield so as to carry out its mission. This is similar to real operations, in which a ship would try ways to go through a minefield with the minimum effort to complete its given task instead of wasting time going back to the starting position and starting all over again. The description of the maneuver is listed in Figure 24.

c. Finite-width Minefield

The minefield is finite in the x-direction. In the NDLAM model, the minefield is infinite in the x-direction so if the ship returns to the beginning of the field it chooses another initial position such that its future path will not intersect with a previous path. The ship can do this an infinite number of times until it either gets across the field or gets sunk. In the EMM, when the ship is blocked the ship will not necessarily return to the beginning of the field to try an independent path. Instead, it will search for a path around the blocked squares. However with finite minefield size, it is now possible that a path cannot be found.

2. Analytical Model

Due to the complexities of the maneuver, a full analytical model could not be developed for this model. However, in Chapter V we presented a special case of this model in which the corresponding analytical model has been developed, and the analytical model results were used to partially validate the simulation. The model in Chapter V represents portion of the maneuver for EMM.

Since the underlying simulation model for the EMM model is essentially the same as the special case for the forward-looking portion and similar to the NDLAM model for the back-stepping portions, the results obtained by the EMM simulation are partially validated by the combination of these two models.

C. METHOD OF SIMULATION

The simulation is very similar to that for the NDLAM model, except that we expanded the pseudo code in Figure 4 for Chapter III to include the variation of the minefield patterns shown below:

1. Main Simulation

First, generate 1000 minefield patterns with specified rate of NOMBOs and mines and store in an array. The total number of objects is drawn independently from a binomial distribution. The process of drawing the numbers and sowing them into the minefield is described in Figure 5 from Chapter IV and also in Chapter V. Illustrated in Figure 23, there are 3 major loops that control the execution of the simulation. The 1st loop varies the minefield pattern being used. The 2nd loop varies the initial starting position from leftmost, center and rightmost position relative to the minefield, using different random numbers. The 3^{rd} loop changes the Probability of Detection (pd) in steps from 0.0 to 1.0 so as to estimate the probability of a successful transit due to the maneuver if the sonar sensor becomes increasingly effective. For example, if a sensor is not effective, i.e. pd is very low, the effects of the maneuver will not be apparent on the probability of safe transit because the vessel will not take avoidance actions when it cannot detect anything in front of it.

Using the analogy of the blind man, if his probing stick is short (analogous to a low pd), he would not be able to reach very far ahead or low enough to detect objects, so the chances of missing something is very high. On the other hand, if his stick were long (higher pd), he would be able to detect objects further ahead so that he would be able to take the appropriate avoidance actions. The pd is varied in steps of 0.1 for the third loop. For each replication of the simulation, the vessel will start from an initial position that is fixed, and makes its way to the end of the minefield unless it gets blocked completely, in which case there is no way across, or gets blown up if it goes on top of an undetected mine. The resultant pseudo code is very similar to that of the NDLAM simulation model. The program then records the results for each replication to obtain the required statistics.

In this model, random number streams are not common from pd to pd because they would be correlated across 2000 patterns as well

```
1.        For each minefield pattern, perform the following:

2.        For each starting position, perform the following:

3.        Repeat for each Pd used, in increments of 0.1 (from 0.0 to 1.0) the following steps.

{

          Get vessel's starting position from array and place it there
          Do {

                    Check the next square ship is heading to for mines, nombos or
                              false alarms
                    If (no detection)
                              Move ship to new square
                              If there is a missed mine
                                 Attempt fails
                                  Mine detonates, and the ship is destroyed
                    Else
                              Determine next position to go to

          }
          While (ship has not crossed finish line and no detonations occur)
          Record failure or success and other statistics
          Reset variables and transit history on minefield.
}
```

Figure 23. Pseudo Code for Main Simulation Process for EMM model

D. ENHANCED MANEUVER

1. Algorithm for Enhanced Maneuver

Next we describe the maneuver that is an enhancement of the NDLAM maneuver. As we saw from the previous section, the general simulation process is similar to that of NDLAM. The main difference is in how the next position is determined. Building upon the pseudo code described in Chapter 3 Figure 10, the pseudo code on how the next position is determined is shown:

Determining next position to go

Get Information

1. Get current position of ship in minefield; say (x,y)
2. Get information from previous determination of the presence or absence of objects in the squares ahead of ship.

Conditions for blocked square

3.. A square is blocked if an object is detected or a false alarm occurs for longer than the classification time

Checking available pathways for all 5 directions

4. If the square (x,y+1) is not blocked, the ship moves into the square and next square to check is (x,y+2) if the ship survives the move
5. If square (x,y+1) is blocked, set the next square to check is (x-1,y+1)
6. If the squares ahead and left-ahead blocked, set the next square to check is right-ahead (x+1, y+1)
7. If the front 3 squares are blocked, check the left square i.e.. (x-1,y). If the square is not blocked, the ship moves into the square. If it survives the move, the next square to check is (x-2, y+1) since (x-1,y+1) was found blocked earlier on.
8. If the front 3 squares and the left square is blocked, check the right square (x+1,y) and perform similar checks to those of the left square.

Handling backtracking

9. If all 5 directions are blocked i.e. (front, left-ahead, right-ahead, left, right blocked)
   - Backtrack one step backwards i.e. go back to the square it came from before the current square.
   - Set the direction it came from as blocked to prevent looping
   - Check for directions from the present square which are not blocked and not transited before and set the next square to check in that direction
   - If there are multiple directions that are not explored or not blocked, follow the sequence as set from steps 4 to 8, i.e. check ahead first, followed by left-ahead etc.
10. Return to main loop of main simulation to pursue the available direction

Figure 24. Pseudo Code for determining next position to go for EMM model

2. Comparison Of EMM's Maneuver To Robotic Mice Maneuver

This maneuvering procedure is similar to the procedure being used by computer mice trying to find their way around a maze using a "wall-tracing" algorithm. (Allen 2003) Indeed one can view the ship as a similar construct being "trapped" in a maze of mines and NOMBOs once it enters the minefield and tries to find a way out. The difference is that in a maze there is only one way in and one way out; whereas in the minefield there are often multiple ways in and multiple ways out, depending on where the ship starts its transit. However, this is not necessarily true when the minefield density of NOMBOs and objects gets high enough such that there is a "wall" of mines and NOMBOs which cannot be penetrated regardless of the starting position of the vessel. In such a case, the ship will fail in its mission to safely cross the minefield , and as would the robotic mouse.

The second difference between this maneuver model and the robotic mouse is the treatment of walls. In the case of the robotic mouse, the walls are fixed from the beginning and the mouse will trace a path around the wall. It may or may not remember where it has been, depending on the model representation. For the vessel transiting the minefield, there are initially no walls, so to speak. The walls only appear as the ship transits the minefield. In the current implementation, the vessel would check the square ahead before it moves into it. If there is an object and it gets detected, the vessel will label the square as blocked. The other cases where the ship would label a square as blocked would be when a false alarm occurs and misleads the system into indicating there is an object there, or when the ship backtracks from that square. The ship also checks whether the square has been transited before and from which direction. In those cases where the direction to that square is blocked, the vessel builds a "wall" in that direction in the current square to prevent itself from going there again. All the information on what squares have been examined and from what direction the squares have been examined is subsequently saved.

3. Comparison of EMM maneuvers to real operations.

The ship "remembers" where it has been by maintaining a history track of the path it has traversed, and the positions of the objects it had detected. This is similar to what a minehunter would do, i.e., this information would all be plotted onto a grid usually displayed in the screen of the Mine Information System or MEDAL system deployed onboard the USN MHCs (mine hunters, coastal) The information gathered for this swept or cleared channel can then be relayed to the main fleet, through MEDAL or GSYS. In the case of a lead-through operation, where a lead ship, usually a MCMV, would lead a ship or a group of ships going through a minefield, the lead ship would transmit information about the path beforehand and, while transiting the path, signal its course, speed and precautions to the ships behind. For this to be done, the lead vessel needs to know where the dangers are and the boundaries of the safe channel so that it can transit safely. For the vessels following behind, the assurance is that they are following the path given by the minehunter, with the added assurance that the way ahead is somewhat safer because the lead ship has already passed through it. This however, does not take into account the possibility of ship counters in the mines: with this capability the mine could be pre-programmed to activate after it detects the n^{th} ship passing through.

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VII. SIMULATION OF EMM MODEL AND RESULTS

A. INTRODUCTION

This chapter will present the simulation results from the EMM model and provides an analysis and comparison of the results with that from the NDLAM model. Using the estimated probability of safe minefield transit as the measure of performance, simulations were performed for three cases: the effects of starting positions on safe minefield transit, the combined effects of mines and NOMBOs, the combined effect of mines, NOMBOs and false alarms. An analysis of the results is also presented.

B. CONDUCT OF SIMULATION

The simulation for the EMM model was set up as described in Chapter V and similar to that described in Chapter VI. The simulation was conducted with the same parameters as with the NDLAM model. In particular, the expected density is 5 mines and 10 NOMBOs per square nautical miles. In addition, the simulation was conducted first without false alarms. Different minefield patterns were drawn independently from binomial distributions to determine the total number of objects, mines and NOMBOs with the same parameters of expected number of objects, mines and NOMBOs; in this case 5 mines and 10 NOMBOs per square nm. Random numbers are independent for each pd.

The length and width of the minefield was 5nm by 5nm. A total of 22000 replications were performed (2000 different minefield patterns x 11 steps of varying probability of detection (from 0 to 1.0) x 1 replication per pd). Furthermore, on each replication of the simulation, the vessel is transiting a 50-square long minefield performing checks for each square. Due to the limitation in the memory size of the system, it was not possible to conduct 10000 iterations per pd used as in the NDLAM model. 2000 independent minefield patterns will result in 2000 independent observations. Each replication is an attempt by the vessel to transit the minefield; the replication ends with an outcome of either mission success, mission failure due to detonation of undetected mines, or mission failure due to blocked paths.

At the end of the simulation, the probability of a safe minefield transit is estimated for each minefield pattern, and for each pd, and written into a EXCEL file for plotting the graphs. A typical result of the mean number of objects, mines and NOMBOs drawn from a binomial distribution is shown in the next table:

Avg Number of objects	374.56
Std Dev of objects	17.41
Std Err of objects	0.55
Avg Number of mines	125.02
Std Dev of mines	10.99
Std Err of mines	0.35
Avg Number of nombos	249.54
Std Dev of nombos	14.84
Std Err of nombos	0.47

Table 10. Average number of total objects, mines and NOMBOs

C. EFFECT OF STARTING POSITIONS ON PROBABILITY OF SAFE TRANSIT ACROSS THE MINEFIELD

In Figure 25, the results from Table 11 are plotted, and the graphs of the results of the 3 initial positions show that the results from the left or right positions are generally contained within the approximate 95% confidence interval of the center position, indicated by the y-error bars and CI lines. As the curves are overlapping each other's 95% confidence intervals, these results indicate that there is little significant impact on the probability of safe transit from a vessel's initial starting position to cross the minefield for these parameters values.

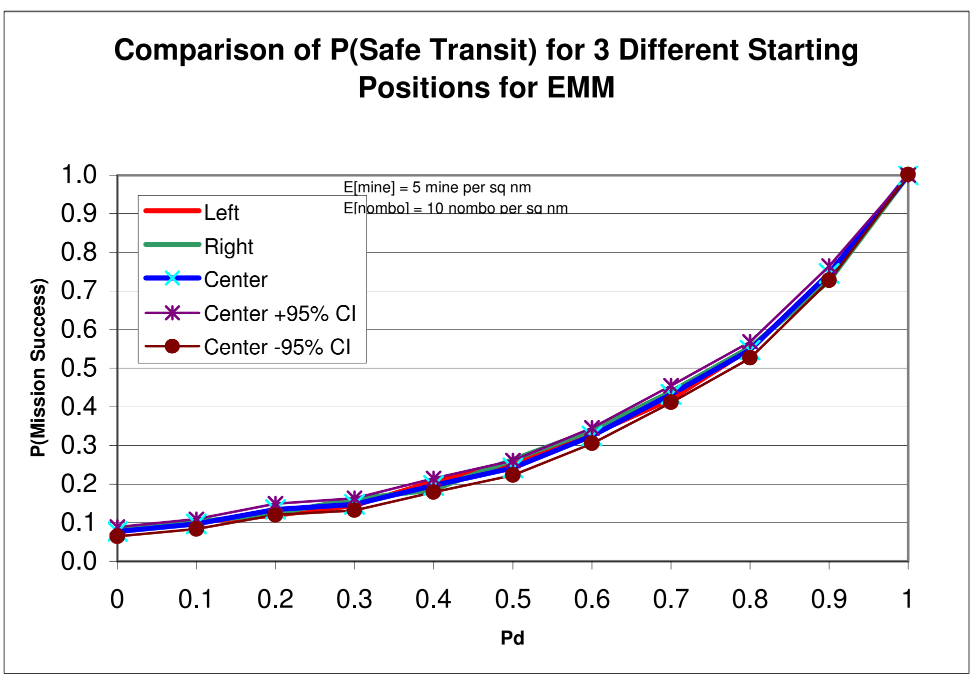

Figure 25. Comparison of P(Safe Transit) for 3 different Starting Positions for EMM

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
P(safe transit) Leftmost	0.076	0.101	0.119	0.146	0.207	0.252	0.329	0.420	0.550	0.739	1.000
Std Error	0.006	0.007	0.007	0.008	0.009	0.010	0.011	0.011	0.011	0.010	0.000
P(safe transit) Center	0.076	0.097	0.135	0.148	0.197	0.242	0.325	0.433	0.547	0.746	1.000
Std Error	0.006	0.007	0.008	0.008	0.009	0.010	0.010	0.011	0.011	0.010	0.000
P(safe transit) Rightmost	0.083	0.101	0.125	0.161	0.186	0.264	0.339	0.439	0.554	0.727	1.000
Std Error	0.006	0.007	0.007	0.008	0.009	0.010	0.011	0.011	0.011	0.010	0.000

Table 11. Results for Effects of 3 Starting Positions on safe minefield transit.

Since the results indicate that the probability of safe transit has no strong dependence on a particular starting points, it was decided that additional simulations be performed using the center position only.

D. EFFECT OF MINES AND NOMBOS ON EMM

A rate of 5 mines and 10 NOMBOs per square nautical mile was used to for comparison to the results of the previous studies in Chapter VI. False alarms are not included in this portion of the simulation. Table 12 shows the results of the simulation for these parameters.

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Average P(safe transit)	0.074	0.088	0.120	0.156	0.196	0.246	0.342	0.430	0.571	0.746	1.000
Standard Deviation	0.261	0.283	0.325	0.363	0.397	0.430	0.474	0.495	0.495	0.436	0.000
Standard Error	0.006	0.006	0.007	0.008	0.009	0.010	0.011	0.011	0.011	0.010	0.000

Table 12. Results from EMM using simplified model

The results were obtained in the following manner: For each of the 2000 minefield patterns the vessel goes through starting in the middle of row 0. The simulation estimates the probability of safe transit for varying levels of probability of detection. For each pd, the outcome is either mission success or mission failure. Thus for each position, the estimated probability of mission success for that pd is the sum of the total successes with that pd averaged over the 2000 minefields to obtain the results in Table 12. The results show that the estimates of the probability of successful transit are quite dependent on the probability of detection. Since there are no correlated random numbers, the standard errors are calculated using the usual formula with 2000 independent observations. The overall trend shows that the probability of safe minefield transit increases with pd, which is similar to the results obtained from the NDLAM model. When pd is 1.0, the vessel will always be able to detect any object in a square, and always succeeds in finding a way through the minefield provided there is a path through the field.

E. COMPARISON BETWEEN NDLAM AND EMM RESULTS

The simulation results from the NDLAM and EMM (without false alarms) models are compared in Table 13. From Section C, we have determined that the results are not dependent on the starting position of the minefield, so we will only compare the NDLAM results with the results obtained from the center position in the EMM to simplify comparisons. They indicate that the results obtained were similar, as the EMM results were mostly contained within the 95% confidence interval of the NDLAM results.

NDLAM Simulation Results

Probability of Detection	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P(Mission Success)	0.080	0.092	0.116	0.156	0.202	0.256	0.329	0.420	0.560	0.732	1.000
Std Error of P(Success)	0.003	0.003	0.003	0.004	0.004	0.004	0.005	0.005	0.005	0.004	0.000
+95% CI	0.085	0.098	0.122	0.163	0.210	0.264	0.338	0.429	0.570	0.740	1.000
-95% CI	0.075	0.086	0.110	0.149	0.194	0.247	0.320	0.410	0.550	0.723	1.000

EMM Simulation Results

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Average P(safe transit)	0.074	0.088	0.120	0.156	0.196	0.246	0.342	0.430	0.571	0.746	1.000
Standard Deviation	0.261	0.283	0.325	0.363	0.397	0.430	0.474	0.495	0.495	0.436	0.000
Standard Error	0.006	0.006	0.007	0.008	0.009	0.010	0.011	0.011	0.011	0.010	0.000

Table 13. Tables comparing NDLAM and EMM simulation results using expected number of 5 mines and 10 NOMBOs per square nm.

G. DISCUSSION

1. Initial Starting Positions in a Minefield

The results in Figure 25 show that the starting positions of the vessel when transiting the minefield does not have a significant impact on the probability of safe transit. The plots for the probability of safe transit for the leftmost, center and rightmost positions are close to one another, mostly within the 95% confidence interval of the center curve. In the analytical model of the 3 square wide case, there is some difference, but it was not obvious in the simulation. From the results in Figure 21, the differences are very small. This means that there is no perceived advantage when transiting a minefield to start at the edges or in the center because the probability of mission success is similar for all three positions, as inferred from the results for these parameters. This may have doctrinal impact on minefield transits; however more investigation must be done.

2. EMM and NDLAM Simulation Results

The EMM simulation result show that the probability of safe minefield transit increases exponentially with increases in pd, which is similar to that from the NDLAM model. For the parameters chosen the results for the two models are comparable. It was expected that the EMM would have higher probabilities of success because of the enhanced maneuvers and the shorter expected time spent in the minefield but the results seem to indicate the models are similar. The reasons were found to be as follows:

a. Different Outcomes for Measure of Performance

On review of the models, it was found that the Measure of Performance for the two models are different. In the NDLAM model, the possible outcomes for each attempt in minefield transit were only two: success, or failure through detonation of a missed mine. The vessel can backtrack as many times as it wants until an outcome resulting in either failure or success is achieved. For the EMM model, the outcomes are mission success, mission failure due to encounter with an undetected mine, or mission failure because there is no path at all to the end of the minefield. So although the range of values in the results appears to be of the same magnitude, the outcomes may be different. A more appropriate measure would be the probability of transit success instead of safe transit because in the EMM there would be instances where the vessel would be safe, i.e. not destroyed by mines but unable to complete the mission due to blocked paths. This will be elaborated in detail in the next section.

b. Mine Avoidance Problem/Paradox

In Kim's research (Kim 2002), it was discovered that the probability of safe transit actually decreased as the probability of detection increases for some parameter values. It was ascertained that as the sensor performance increases, the vessel becomes better at detecting objects ahead, and as a result takes more avoidance actions that would result in more backtrackings to the beginning of the minefield to attempt another transit through the minefield from another position. The resultant increase in the distances traveled in the minefield increases the probability of the vessel being blown up, thus lowering the probability of safe transit. However, this effect was not apparent in

Figure 24 for the parameters used. Therefore, to enable a compatible analysis, the parameters used by Kim's thesis are scaled to the NDLAM and EMM models by using the same numbers for nombo and mine density per square nm. The analytical results are computed using the NDLAM analytical model and the simulation results are obtained by using the same parameters for the EMM. In this simulation all replications are independent between pds as correlated random number streams were not used. Expected number of mines and NOMBOs used were 1 mine and 48 NOMBOs per square nautical mile. The minefields were generated using the procedures described in Figure 16 on the drawing of mines, NOMBOs and objects from Binomial Distributions and the objects sown into the minefield using the process described in Figure 6. The estimated probabilities of safe transit are displayed in Figure 26.:

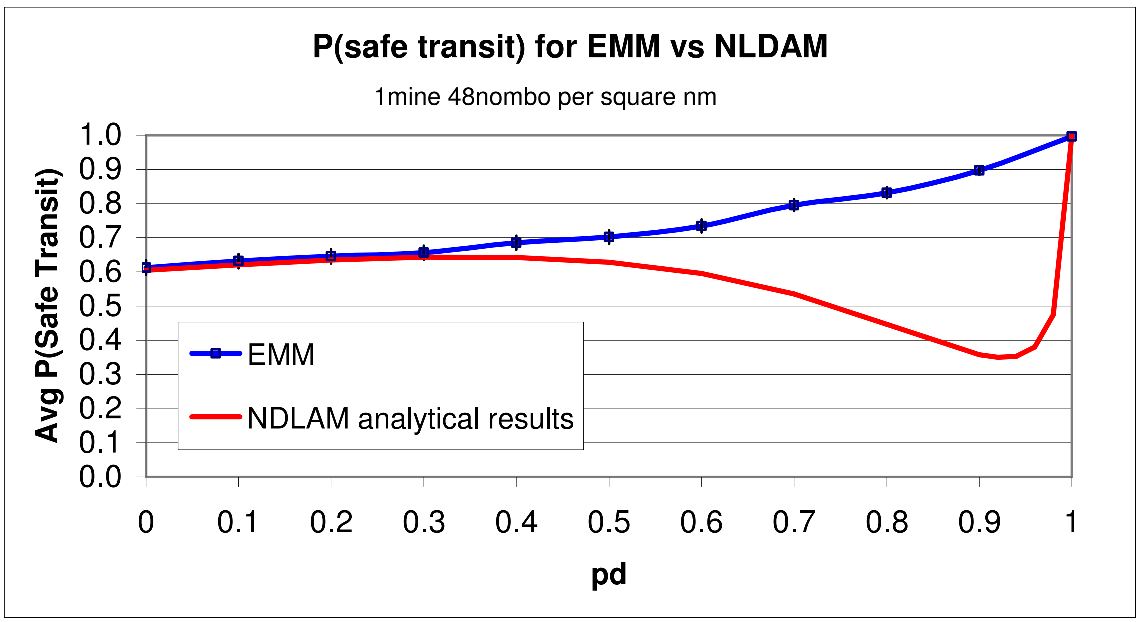

Figure 26. Comparison of EMM vs. NLDAM (Expected No of mines =1, Expected No of Nombos = 48)

The analytical result of the NDLAM indicates that as pd increases from 0.3 to 0.9, the average probability of safe transit actually decreases from 0.64 to 0.35, before increasing again when pd increases from 0.94 to 1.0. One of the possible reasons why this paradox exists i.e. the probability of safe transit decreasing when pd is increasing, could be explained that at low levels of pd (from 0.0 to 0.3) and object densities the effect of maneuvers on safe transit was not strong because the probability of the vessel detecting something is small at those pd levels, and so the corresponding need to take avoidance actions is small as well. Therefore the corresponding results for the NDLAM and EMM appears to be the same for those density levels.

As pd increases up to 0.3, there is some advantage gained by the maneuvering actions for the NDLAM model as well. However, from pd=0.4 onwards, the vessel in the NDLAM model would backtrack more often i.e. return to the beginning of the field and retry the transit again along a nonintersecting path. Since it spends more time and covers more distance in transit, the probability of the vessel being blown up increases, resulting in lower probability of safe transit. On the other hand, the EMM does not show this behavior using these parameters and still increases with pd because it is better able to find a successful path with its maneuvers. This behavior, in which the probability of mission success decreases with intermediate values of pd was only exhibited when the object density becomes sufficiently large for the minefield. The previous mine and NOMBO density used were 1 mine and 48 NOMBOs per square nautical mile (nm). When the expected number of NOMBOs per sq nm is increased further from 48 to 55, the probability of safe transit tapers off and decreases to 0.64 (standard error of 0.01) when the pd increases to 1.0. See Figure 27 for the graph and Table 13 for the results.

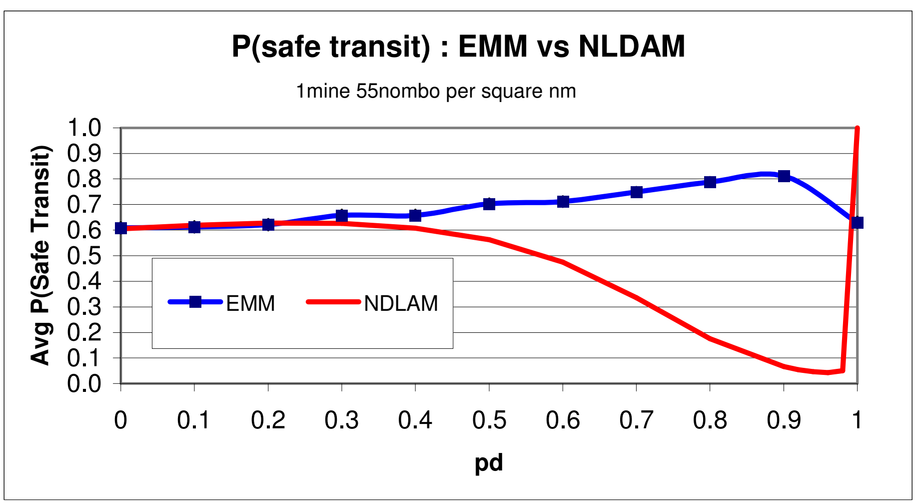

Figure 27. Comparison of EMM vs. NDLAM Simulation Results with Higher Nombo Density (Expected mines per square nm =1, Expected NOMBOs per sq nm = 55)

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Average P(safe transit)	0.590	0.631	0.629	0.652	0.671	0.699	0.713	0.776	0.783	0.827	0.644
Standard Deviation	0.492	0.483	0.483	0.477	0.470	0.459	0.452	0.417	0.412	0.379	0.479
Standard Error	0.011	0.011	0.011	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.011
CV	1.9%	1.7%	1.7%	1.6%	1.6%	1.5%	1.4%	1.2%	1.2%	1.0%	1.7%

Table 14. Results Exhibiting Effects of Blockages on P(Safe Transit) (expected mine and NOMBO density 1mine and 55 NOMBOs per sq nm)

The results found in Table 14 indicate that if the total object density gets sufficiently high and when the sensor becomes very effective (in this scenario, when pd=1.0), the probability of mission success actually drops to 0.64, and the EMM model will not achieve a 100% success rate. This behavior is due to the minefield being finite, in which the density of mines and NOMBOs become so much that the ship gets blocked more often, resulting in more mission failures due to blockages than due to missed mines as the vessel's sensor become more effective. The proportion of blockages given mission failure is shown in Table 15 and plotted in Figure 28 as follows:

Probability of Detection	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
% of blocked transits over total failures	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0265	1.0000
Standard Error	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0016	0.0108
CV	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	5.9%	1.1%

Table 15. Percentage of Blocked Transits Over Total Failures

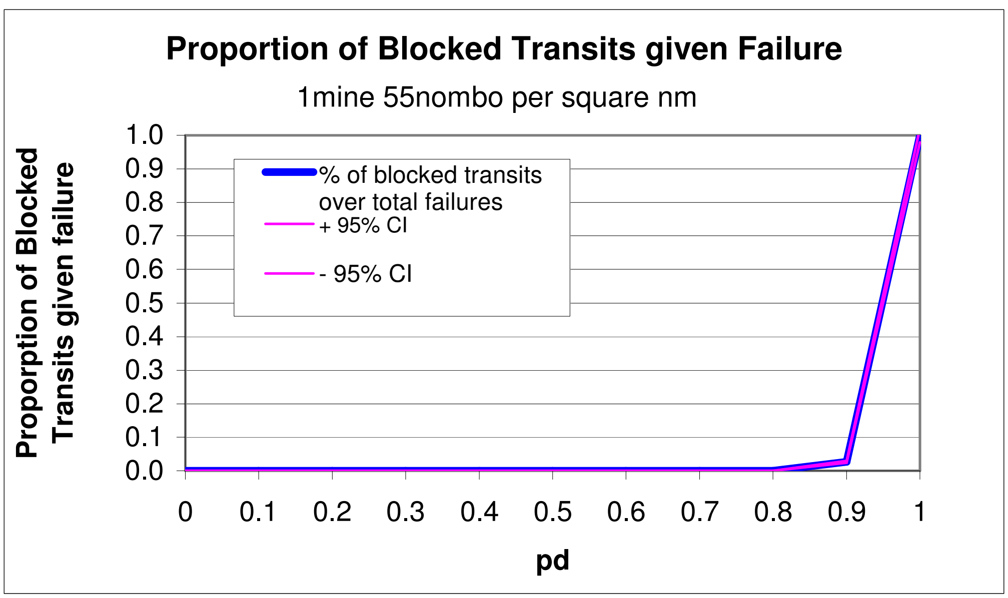

Figure 28. Proportion of Blocked Transits Given Mission Failure

The results indicate that with the given NOMBO and mine density, all the mission failures when pd = 0 to pd = 0.8 are attributed to undetected mines as there were no failures that were due to blocked transits. However, when pd is > 0.8, the vessel would on average take more avoidance actions as its sensor becomes more effective, and becomes increasingly frustrated by blocked pathways as it tried to cross the field. When pd is less than 1.0, there are instances when the vessel would go over a NOMBO unknowingly and will be able to still achieve mission success. However, when pd becomes 1.0, the vessel becomes fully aware of all the objects in front of it, and because of the policy that it will avoid all objects, NOMBOs or mines, the probability of mission failure increases when the patterns are formed in such a way that there is not a single free square that the vessel can cross over. Figure 27 shows that with pd =1.0, all the mission failures were due to blocked paths instead of being destroyed by missed mines since the vessel will not enter any square with an object and miss any mines with a "perfect" sensor.

Drawing the analogy of the blind man crossing a field again, when his stick is short (low pd), he would sweep the stick around less effectively and may not touch the ground most of the times. In this manner, he would not be able to detect most objects near the floor and be only able to detect those that the stick can reach. If there is a ball (representing a NOMBO) in front of him on the floor, his stick may not reach it in time when he goes over it. The end result is that he may just step over it or kick the ball away as he crosses it, and no harm will come to him. Now, if he has a very long stick (high pd) and he can use it very effectively, he may detect the ball from afar. However, if his policy is to avoid all objects in his path, he would have to circle around the ball (assuming a big ball) so as to search for a clear path for him to cross. If he was going through a playground and there are lots of balls lying around him, there will be situations when he will not be able to cross the area at all, and would have to find other means to cross the field instead. If he is able to differentiate between a harmless ball and huge mousetrap (where his foot may be caught and hurt), he may choose to step over the ball safely and proceed along instead, and not be disturbed if there are too many balls lying around. Similarly, if a mine avoidance sonar is able to classify objects accurately, we would expect better results.

Looking at the NDLAM results in Figure 26, we see that there is the probability of safe transit increases to 1.0 when pd becomes 1.0, despite that it was 0.05 when pd = 0.98. This result is explained by the fact the vessel keeps backtracking infinitely and the minefield keeps being redrawn whenever there is a blockage, such that there will always be a way to cross the minefield safely each iteration as the simulation keeps trying. This would not be realistic in real life.

In conclusion, we see that the results from the EMM differ from the results in the NDLAM when the expected mine and NOMBO density is high, e.g. when there is an expected number of 1 mine per sq nm and 55 NOMBOs per square nm, and when the pd is more than 0.4 for this density. At lower object densities, the effect is not apparent. If the MAS modeled in the EMM is able to classify objects accurately, then the result would be expected to differ, and more research in this area is recommended.

4. Effectiveness of EMM Model on Minefield Transit

Assuming that a typical mine-avoidance sonar system would recommend the avoidance maneuvers as described in the EMM model and that the vessel would carry out the maneuvers as recommended, we can utilize the results below to provide some answers to the basic question on the effectiveness of the mine avoidance sensor on minefield transit. The following graph was obtained to illustrate the probability of safe transit from selected varying mine and NOMBO number density per square nm in the minefield.

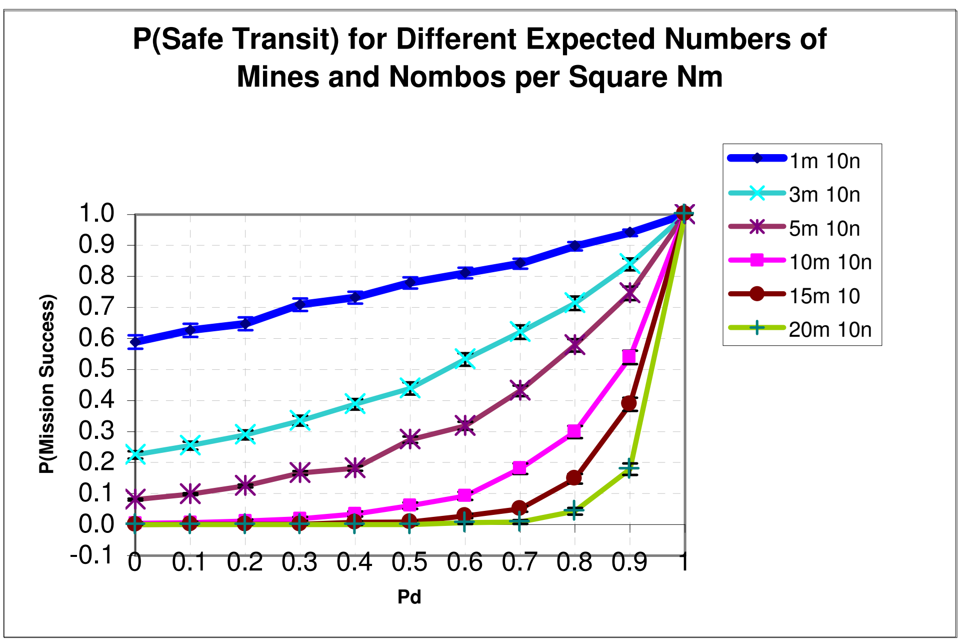

Figure 29. Simulation Results For EMM For Various Mine And Nombo Density

Probability of Detection	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
1m 10n	0.589	0.627	0.648	0.709	0.732	0.779	0.811	0.841	0.897	0.941	1.000
Std errors	0.011	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.007	0.005	0.000
3m 10n	0.225	0.255	0.290	0.336	0.389	0.439	0.533	0.621	0.713	0.839	1.000
Std errors	0.009	0.010	0.010	0.011	0.011	0.011	0.011	0.011	0.010	0.008	0.000
5m 10n	0.081	0.098	0.125	0.166	0.181	0.275	0.319	0.432	0.578	0.746	1.000
Std errors	0.006	0.007	0.007	0.008	0.009	0.010	0.010	0.011	0.011	0.010	0.000
10m 10n	0.004	0.006	0.012	0.019	0.034	0.061	0.092	0.180	0.299	0.539	1.000
Std errors	0.001	0.002	0.002	0.003	0.004	0.005	0.006	0.009	0.010	0.011	0.000
15m 10n	0.000	0.002	0.001	0.002	0.007	0.008	0.028	0.051	0.147	0.388	1.000
Std errors	0.000	0.001	0.001	0.001	0.002	0.002	0.004	0.005	0.008	0.011	0.000
20m 10n	0.000	0.000	0.000	0.000	0.001	0.001	0.006	0.009	0.043	0.179	1.000
Std errors	0.000	0.000	0.000	0.000	0.001	0.001	0.002	0.003	0.006	0.010	0.000

Table 16. Tabulated results for EMM for Various Mine and Fixed NOMBO Density

From Figure 29 or Table 16 we see that for a mine density of 1 and 10 NOMBOs per square nm, a system with an average performance to render a pd of 0.5 to 0.6 would have about 78-81% chance of transiting the field safely without being blocked or blown up. Holding the mine density per square nm fixed and varying the NOMBO density, the results in Figure 30 indicate there is a general effect of NOMBO density on probability of minefield transit which becomes pronounced when NOMBO density becomes 55.

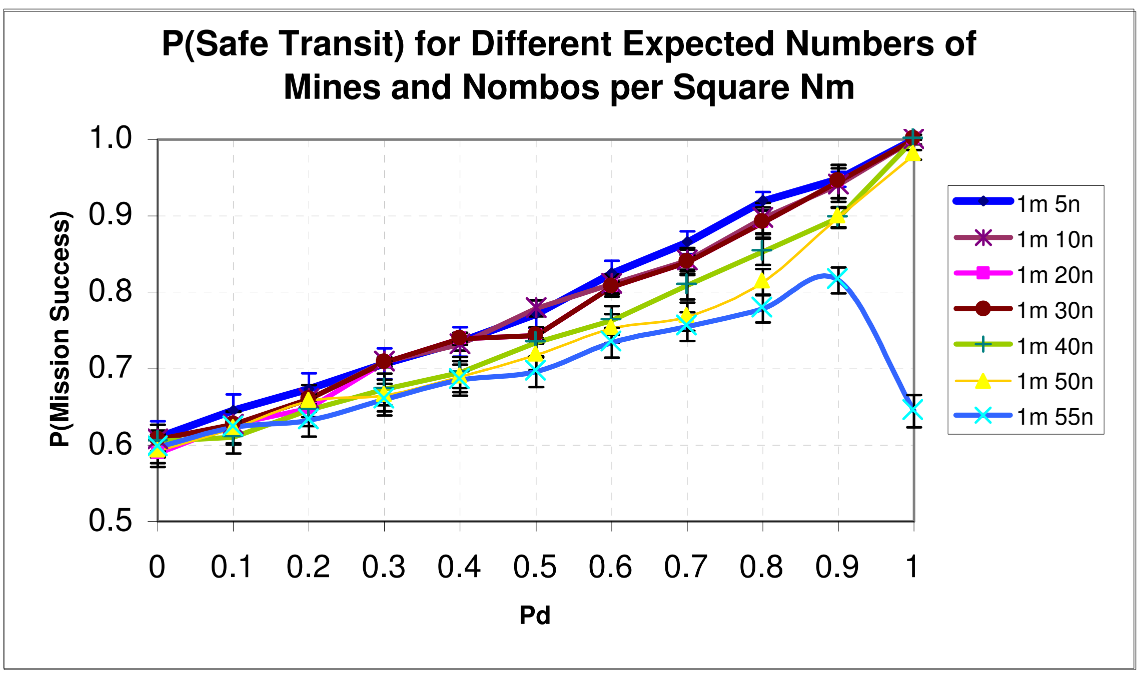

Figure 30. Effect of NOMBO density on Probability of Safe Transit

Probability of Detection	0.000	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
1m 5n	0.610	0.646	0.674	0.707	0.735	0.772	0.825	0.865	0.920	0.948	1.000
Std errors	0.011	0.011	0.010	0.010	0.010	0.009	0.009	0.008	0.006	0.005	0.000
1m 10n	0.608	0.627	0.660	0.709	0.732	0.779	0.811	0.841	0.897	0.941	1.000
Std errors	0.011	0.011	0.011	0.010	0.010	0.009	0.009	0.008	0.007	0.005	0.000
1m 20n	0.589	0.627	0.648	0.708	0.739	0.744	0.807	0.839	0.892	0.945	1.000
Std errors	0.006	0.007	0.007	0.008	0.009	0.010	0.010	0.011	0.011	0.010	0.000
1m 30n	0.608	0.627	0.660	0.708	0.739	0.744	0.807	0.839	0.892	0.945	1.000
Std errors	0.001	0.002	0.002	0.003	0.004	0.005	0.006	0.009	0.010	0.011	0.000
1m 40n	0.605	0.611	0.646	0.673	0.695	0.734	0.763	0.809	0.853	0.898	1.000
Std errors	0.000	0.001	0.001	0.001	0.002	0.002	0.004	0.005	0.008	0.011	0.000
1m 50n	0.593	0.622	0.658	0.665	0.690	0.718	0.753	0.768	0.813	0.899	0.980
Std errors	0.011	0.011	0.011	0.011	0.010	0.010	0.010	0.009	0.009	0.007	0.003
1m 55n	0.598	0.624	0.633	0.660	0.685	0.696	0.734	0.755	0.779	0.816	0.645
Std errors	0.011	0.011	0.011	0.011	0.010	0.010	0.010	0.010	0.009	0.009	0.011

Table 17. Tabulated Results on NOMBO Density Affecting Probability of Safe Transit

When the NOMBO density per sq nm is doubled to 20 NOMBOs, the probability of safe transit is about the same, i.e. not significantly different. When the NOMBO density is very high i.e. more than 50 NOMBOs per square nm and the mine density is 1 per square mile, the probability of mission success drops to 0.645 when pd is equal to 1.0

Recall that pd is the prevalent probability of detection based on sensor performance interacting with the environment. A powerful sonar may not achieve this effective pd if the sea bottom is composed of hard shells, rocks where there is strong reverberation, and noise/reflections from the bottom that would affect sonar performance, or if the bottom is undulating and rough that would present features that could potentially be mistaken for NOMBOs. Other factors would include the speed the ship is traveling, the sound velocity profile of the water column, etc. that would affect the performance of the system. On the other hand, a less powerful sonar system may be experiencing ideal environmental conditions like uniform sound velocity profiles in the water column, flat smooth bottoms which are uncluttered with other objects etc. In this scenario the system may easily achieve high pds, but as the sensor becomes more sensitive, the probability of false alarms occurring becomes higher.

Going back to the graph in Figure 29, if the number of mines and NOMBOs is high at 20 mines and 10 NOMBOs per square nm , the probability of success is very low, practically close to 0 until pd increases to 0.7 onwards. This means that with this combination of mines and NOMBOs, transiting the minefield is not really feasible, and the vessel would be better off finding another area to transit, or that mineclearance /

minesweeping operations is really required in order to go through this minefield, and mine avoidance alone in this case would not be useful.

These results indicate that the effect of mine density have stronger effect on probability of safe transit than does the NOMBO density. For every increase in a mine per square mile, the whole curve for the probability of safe transits changes by a lot. The results from Figure 30 indicates that NOMBO density has the effect of lowering probability of safe transit as well, but to a lesser extent as compared to additional mines. However, with very high NOMBO densities e.g. 55 and high pds, it is observed that blocked paths have much stronger effect on mission failures than does encounters with undetected mines in which the ship is killed. This is because with higher pd the chances of the vessel not detecting a mine gets smaller, while the chances of being blocked gets higher due to the increased number of mines and NOMBOs and the conservative avoidance maneuvers taken. For example, an area of one square nautical mile would have about 100 squares, each square is 200 yards in dimension in both length and width. When the NOMBO density is about 50 per square mile, we would start to see mission failure due to blockages when pd is high at 0.8 onwards. These results indicate that for this model and the cases considered the probability of mission success is primarily dependent on the number of mines present in the field, followed by the number of NOMBOs.

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VIII. CONCLUSIONS AND RECOMMENDATIONS

A. CONCLUSIONS

The thesis sets out to model and simulate the discrete minefield version of the Neighboring Diagonal Local Avoidance Model (NDLAM) so as to validate and extend the results obtained from an earlier thesis (Kim 2002). Then a newer model, the Enhanced Minefield Maneuver (EMM) model, is implemented and simulations conducted to investigate whether an improved representation of mine avoidance tactics would lead to larger probabilities of safe minefield transit than those found in Kim (2002). Since an analytical model for the EMM was not attempted because of the complicated backtracking component of the maneuver, a special case of the EMM was developed and simulated to provide both an analytical basis and partial validation of the corresponding simulation. Various results were compared for the NDLAM model against the EMM model, and also against the special case. From the results, some remarks on mine avoidance tactics are then suggested.

B. RECOMMENDATIONS FOR MINE AVOIDANCE TACTICS

It must be noted that the present mine avoidance study is focusing only on NOMBOs and mines, and not looking at environmental conditions, types of bottom composition, ship counts etc. Also, the assumptions here are that the vessel has an accurate positioning system that is able to mark the locations of the objects accurately and also its own location, and it is also able to execute maneuvers well enough so that no mines are detonated in the avoidance process. Under these conditions, the question on the effectiveness of mine avoidance sonars can be partially answered by looking at the Figure 26 in accordance with the type of operating areas and the desired probability of safe transit:

1. Open Waters (typically expected density: 0-5 NOMBOs square mile)

As mentioned in the previous section, the results indicate that NOMBO density will have a significant effect on the probability of safe minefield transit only when the area is so littered with objects that almost half of it is covered. In open waters, this is usually not the case. Assuming the expected number is 10 NOMBOs and 1 mine per square nm in the field, the probability of mission success increases from 0.6 when pd = 0 to 1.0 when pd = 1.0. The mine avoidance sensor (MAS) is therefore most effective here when the NOMBO and mine density is relatively low. If, however, the mine density is 1 and the NOMBO density is 55, then the average probability of mission success will still increase with pd until it reaches close to 1.0, where it will be lower because the avoidance maneuvers are often not possible. The use of a finite-width minefield model is still appropriate to open waters because even though in theory a ship can travel anywhere when it is navigable, traditional ship routes are usually traversed for expediency purposes, and we can make the finite-field large but not truly infinite. Minefield planners will typically not mine in open waters due to the large number of mines required and the available sea room for maneuver. However, this situation will apply when naval combatants need to transit some area of waters either on the surface or underwater, which is known to be mined from intelligence. Mines that are effective in open waters are typically moored mines because of the depth, so the MAS would be useful for vessels like the submarines transiting submerged in the open seas.

2. Coastal Waters (typically 5-10 NOMBOs per sq nm, possibly 1-3 mines per sq nm)

Coastal or littoral waters tend to be very shallow, so the type of mines to be encountered would include moored and ground mines. There is also much more traffic plying such areas, and NOMBO density is higher due to historical dumping of objects overboard. The possibility of higher mine densities increases along the coastal waters as these waters are targets for both defensive and offensive mining operations. In this case, the effectiveness of the MAS would be more limited due to the need to avoid more objects. Assuming 3 mines and 10 NOMBOs per square nm with a field size of 5nm by 5nm, from Figure 28 the probability of mission success is about 50-60% with an effective pd of 0.6 to 0.7.

3. Approaches to Harbors and Ports (typically >10 NOMBOs per square mile, possibly >3 mines per square nm)

In such areas, mining is highly effective for both offensive and defensive operations. In such cases, it is the norm of the MCM forces to maintain the safety of Q-routes through the conduct of periodic route surveys, which will help ensure a quick path in or out of the harbor areas when the need arises. Channel conditioning operations are typically carried out to remove objects or rubbish from the sea bed along the Q-route, but it is a huge undertaking that requires a lot of resources. Route survey operations are therefore important to maintain their respective mineclearance levels. Using an assumption of 5 mines and 10 NOMBOs per square nm, the vessel can attain a probability of mission success of 30-40% with a pd range of 0.6 to 0.7. Although it is not high, a tactic of maintaining Q-routes for the approaches to harbors and ports during war times, combined with the use of organic mine avoidance sonars would be useful in achieving better probability of safe transits. It is not recommended that ships be equipped with MAS alone, and procedures for maintaining Q-routes be eliminated as the mission success rate will not be high.

4. Combined Tactics

The use of Autonomous Underwater Vehicles(AUVs) is gaining momentum in undersea warfare. The current use of AUVs, for example, the SAHRV (Semi-Autonomous Hydrographic Reconnaissance Vehicle) is to send them in advance to an area to be surveyed, so that side-scan sonar images can be obtained quickly to help locate possible mines and NOMBOs in an area. A path can then be found by finding the uncluttered areas of sufficient size which the vessels can then go through. A combined tactic would be to equip the vessels with organic AUVs that could be deployed anytime. They can help to search for generally safe routes, which the vessel can navigate with the use of the MAS. Another possible tactic would be to deploy the AUV in front of the transiting vessel like “walking the dog” such that the AUV can “sniff” out potential danger areas and demarcate areas with low object density. Used with such a tactic, a vessel equipped with the MAS would be able to transit the minefield with more safety.

C. RECOMMENDED FOLLOW-ON RESEARCH

This thesis can be used as a precursor and stimulus for further studies of enhanced models for mine avoidance and mine warfare tactics. The ability to differentiate between mines and NOMBOs in classification was not modeled in this thesis so as to mirror the current level of technology in mine avoidance sensors. It is expected that in the future such a capability would be possible, and further investigations into this capability incorporating the enhanced tactics discussed here could be pursued. The ability to detect different objects correctly can also be varied in future as this thesis treats the probability of detection for mines being the same as that for NOMBOs.

Further studies on the effect of additional mine activation capabilities can also be made. One activation capability uses ship counts: the capability of a mine to be able to count the number of ships passing it before detonating. Other activation model enhancements would be to include the probability of the mine detonating due to being laid in the water for too long, due to defects, or because of improper deployment. Other areas could include simulating the capability of the vessel in maneuvering around the detected mines that may trigger the detonations, or having obstacles investigated by AUVs (Autonomous Underwater Vehicles) launched from a Littoral Combat Ship (LCS).

To provide other measures of performance, investigations could also be made on the time spent by the vessel executing the EMM maneuvers in the minefield on delays in maneuvering, object detection/classification, and false target mitigation. The probability of detection could incorporate more parameters affecting it, such as the speed of the ship, the Initial Sonar Detection Range achieved for that day, the type and composition of sea bottom, the type of mines being used, sound velocity profiles affecting sonar performance, and many other factors.

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APPENDIX 1 MINEFIELD OBJECT AVOIDANCE MODEL

A. DESCRIPTION OF MODEL

Previous nomenclature will be used here to provide continuity and basis for comparison. as described in their paper. (Gaver, Jacobs and Pilnick, 2003). The specific model named is the Near-Diagonal Local Avoidance Model (NDLAM), which was a more specific name under the NDLAM. For brevity purposes, we will assume the name of the root model NDLAM from here onwards. We assume that the sensor platform is equipped with a medium-to-high frequency obstacle/mine avoidance sonar, and its immediate mission is to transit a minefield of known dimensions safely, operating singly. The traverser **V**, will cross a minefield of known dimensions, and if uninterrupted will travel a distance L directly across the field. However, if a mine is encountered along the way and is undetected, then **V** is immediately destroyed; the possibility of using ship counts, i.e., that a mine can be run over and fail to detonate but detonate on a later encounter is not considered here. If a mine is encountered and detected, or if a NOMBO or false alarm is encountered and detected, **V** may now execute one of a number of detour maneuvers to avoid the obstacle encountered. In this model[1], **V**, in a square with center (x, y) can, in one maneuver, attempt to go to the square directly ahead, (x, y+1); *or* to the left-forward diagonal square, (x-1, y+1); *or* to the right-forward diagonal square, (x+1, y+1). If V (before maneuvering) detects objects or false alarms in all three squares it will discontinue the current attempt, backtracks and returns to the beginning of the field and starts over on an independent path. These steps will repeat until V clears the minefield successfully or until it encounters a mine and is destroyed without detecting it. The measure of effectiveness in this case would be the probability of safe transit across the minefield, and the expected time duration to traverse the minefield. The analytical model is included as follows:

[1] The analytical model in Gaver, Jacobs and Pilnick 2003 is reproduced here and to show how the discrete simulation will be fashioned after.

B. PARAMETERS FOR ANALYTICAL MODEL

The following parameters are defined:

β=Probability an object is within a square; each square statistically-independently contains an object with probability β, and is empty with probability 1-β. Technically, objects occur as a two-dimensional Bernoulli-trials process, the discrete version of the Poisson process. Square dimensions are essentially governed by *V*'s sensor range of detection, presumed to exceed the lethal radius of the mine. We assume $\beta \le 1$, and generally $\beta << 1$.

p_M =Conditional probability the object in a square is a lethal mine, given there is an object in the square;

δ=Conditional probability the object in a square is detected from a neighboring/contiguous square, given there is an object in the square; this defines the maximum range of *V*'s safe detection;

Let γ be the probability that either there is no object in the next square, so no detection opportunity exists, or the square is occupied, the occupant is not a mine, and it is not detected:

$$\gamma = (1-\beta) + \beta(1-p_M)(1-\delta).$$

(x,y)=center of a square.

C. ANALYTICAL MODEL

Let $\boldsymbol{Y}_i$ be the number of squares traveled in the mission direction during the *i*th successful maneuver.

$$P\{\boldsymbol{Y}_i = 1\} = \gamma\left[1+\beta\delta+(\beta\delta)^2\right] \equiv \alpha \qquad (1.1)$$

Let ϕ be the probability the platform detects objects in all the three squares, and so must return to the beginning of the field. By independence

$$\phi = (\beta\delta)^3 \qquad (1.2)$$

The probability a platform is killed when entering a square is $\kappa_M = 1-[\alpha+\phi] \equiv \beta(1-\delta)\, p_M$.

Let $p(y)$ be the probability that a square a *y*-distance in the mission direction across the field is occupied by the platform during an attempt to travel across the field of length $L \geq y$; by independence

$$p(y) = \left[\gamma\left[1+\beta\delta+(\beta\delta)^2\right]\right]^y \equiv \alpha^y \tag{1.3}$$

Let $\boldsymbol{D}$ be the maximum distance across the field the platform can travel without being killed (by an undetected mine). Then for $y = 1, 2, \ldots$

$$P\{\boldsymbol{D} \geq y\} = \alpha^y + \sum_{k=0}^{y-1} \alpha^k \phi P\{\boldsymbol{D} \geq y\} = \alpha^y + \phi \frac{1-\alpha^y}{1-\alpha} P\{\boldsymbol{D} \geq y\} \tag{1.4}$$

Consequently, putting $y = L$ provides the probability that *V* crosses the field without being killed by a mine:

$$P\{\boldsymbol{D} \geq L\} = \frac{\alpha^L}{1-\left[\dfrac{1-\alpha^L}{1-\alpha}\right]\phi} \tag{1.5}$$

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APPENDIX 2 OUTPUT FROM JAVA FOR SIMULATION

The following is a table illustrating the outputs obtained from the Java program during the simulation runs. This case is specific for Chapter IV, for the basic maneuver example.

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Trial Parameters
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Rate of mines per square mile	: 5.0
Rate of NOMBOS per square mile	: 10.0
Rate of objects per square mile	: 15.0
Length of minefield	: 10000 yards (5 nm)
Width of minefield	: 10000 yards (5 nm)
Area of minefield	: 100000000.0 square yards (25 square nm)
Ship speed (in knots)	: 6
Sensor range	: 200 yard
Time to search each square (hr)	: 0.10
Time to transit each square (hr)	: 0.02
False alarm rate	: proportional to pd using ROC curve and DI = 1.0
No of replications per pd	: 10000

Trial Results ::::::::::::

Probability of sensor detection	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
Avg Actual no of objects in area	377.0	377.0	377.0	377.0	377.0	377.0	377.0	377.0	377.0	377.0
Avg Actual no of mines in area	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0	117.0
Avg Actual no of NOMBOs in area	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0	260.0
Total no of Successes each pd	1120	1380	1700	2159	2705	3423	4279	5573	7390	10000
Total no of Failures per pd	8880	8620	8300	7841	7295	6577	5721	4427	2610	0
Total no of Detonations per pd	8880	8620	8300	7841	7295	6577	5721	4427	2610	0
Total no of False Alarms	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Average no of False Alarms	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Probability of mission success	0.12	0.14	0.17	0.22	0.27	0.34	0.43	0.56	0.74	1.00
Std error of mission success	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Avg no of S Squares Searched	5.7	7.1	8.9	11.4	14.5	18.6	23.7	31.3	42.2	58.0
Avg no of S Squares Transited	5.6	6.9	8.5	10.8	13.5	17.1	21.4	27.9	37.0	50.0
Std Dev of S Squares Transited	15.8	17.2	18.8	20.6	22.2	23.7	24.7	24.8	22.0	0.0
Std Err of S Squares Transited	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.0
Time taken to search S Squares	0.6	0.7	0.9	1.1	1.4	1.9	2.4	3.1	4.2	5.8
Time Taken to transit S Squares	0.1	0.1	0.2	0.2	0.2	0.3	0.4	0.5	0.7	0.9
Avg no of F Squares Searched	15.0	15.3	15.6	15.6	15.6	14.7	13.9	11.2	7.1	0.0
Avg no of F Squares Transited	13.9	13.9	14.1	13.9	13.7	12.8	11.9	9.4	5.9	0.0
Std Dev of F Squares Transited	13.0	13.3	13.6	14.1	14.4	14.6	14.9	14.1	12.4	0.0
Std Err of F Squares Transited	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.0
Time taken to search F Squares	1.5	1.5	1.6	1.6	1.6	1.5	1.4	1.1	0.7	0.0
Time Taken to transit F Squares	0.3	0.3	0.3	0.3	0.3	0.2	0.2	0.2	0.1	0.0
Avg Number of backtracks	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.1	0.2	0.4
Standard Error of backtracks	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Avg Max Distance(rows) traveled	17.6	18.2	19.0	19.7	20.8	21.4	22.7	23.2	24.0	23.9
Standard Error of dist traveled	0.1	0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.3

Table 18. Actual Output of Java Program on Simulation Results

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APPENDIX 3 THE PROBABILITY OF SAFE PASSAGE THROUGH A DISCRETE MINEFIELD THAT IS 3 SQUARES WIDE (IN X-DIRECTION) AND OF INFINITE LENGTH (IN Y-DIRECTION)

By

P. A. Jacobs

A. INTRODUCTION

The analytical model was developed by Professor Patricia A. Jacobs for the purpose of the study and is listed here in its entirety.

B. SET-UP OF MODEL

Label the squares: (L,y)=leftmost square y squares (y-distance) into field; (M,y)=middle square y squares (y-distance) into field; (R,y) rightmost square y squares (y-direction) into field.

Suppose the ship is within the minefield in square (L,z). Let $P_{EB}(D > y)$ be the conditional probability the ship in a edge square (say (L,z)) will travel at least y squares vertically (in the y-direction) without encountering an undetected mine or being completely blocked given the ship has already found square (M,z) blocked.

Suppose the ship is within the minefield and in square (M,z). Let $P_M(D > y)$ be the probability the ship can travel at least y more square vertically without encountering an undetected mine or being completely blocked.

Suppose the ship is within the minefield in square (L,z). Let $P_E(D > y)$ be the probability the ship can travel at least y more square vertically without encountering an undetected mine or being completely blocked given it is currently in an edge square, say (L,z), and square (M,z) has not been looked into by the ship sensor.

C. ASSUMPTIONS

There is at most one object in a square. Whether or not there is an object in the square is independent from square to square. Given an object is in a square the ship detection of the object is independent of the other squares. If the ship enters a square with a mine, the ship is blown up. The ship will not enter a square that contains a detected object. There are no false alarms.

D. MANEUVER RULES

If the ship is in middle square (M,z) , the ship will first try next middle square in y-direction, (M,z+1); if blocked the ship will try the diagonal left square (L,z+1); if both (M,z+1) & (L,z+1) are blocked the ship will try the diagonal right square (R,z). If all three square are blocked, the ship will not go further into the minefield (the ship is completely blocked.)

If the ship is in square (L,z): it will first try (L,z+1); if (L,z+1) is blocked then it will try (M,z+1). If (L,z+1) and (M,z+1) are both blocked, try (M,z) if it has not already been found blocked. If ship can safely enter (M,z) then try (R,z+1). If either (M,z) or (R,z+1) are blocked and squares (L,z+1) and (M,z+1) are blocked then ship is completely blocked. It cannot go further into the field.

If the ship is in square (R,z) the maneuver is similar to that when it is in (L,z).

The ship does not backtrack when completely blocked to attempt to find another path. If the ship is completely blocked, it does not move further into the field.

E. PARAMETERS

β = probability there is an object in the square

p_M =conditional probability there is a mine in the square given there is an object in the square.

δ =conditional probability the object is detected given there is an object in the square.

Let $\gamma = 1 - \beta + \beta(1-\delta)(1-p_M)$ be the probability the ship can safely enter a square.

Let D be the additional distance the ship can travel in the vertical direction until it encounters an undetected mine or is completely blocked. Let X be the current position of the ship.

F. ILLUSTRATION OF MODEL

$$P_M\{D>1\} = P\{D>1 \mid X=(M,z)\} = \gamma\left[1+\beta\delta+(\beta\delta)^2\right]$$

$$P_E\{D>1\} = P\{D>1 \mid X=(L,z),\ \text{sensor has not looked at (M,z)}\}$$

$$= P\{D>1 \mid X=(R,z),\ \text{sensor has not looked at (M,z)}\}$$

$$= \underbrace{\gamma}_{\substack{\text{move}\\ \text{safely}\\ \text{into}\\ \text{(L,z+1)}}} + \underbrace{\beta\delta\gamma}_{\substack{\text{(L,z+1)}\\ \text{is blocked}\\ \text{move}\\ \text{safely}\\ \text{into}\\ \text{(M,z+1)}}} + \underbrace{(\beta\delta)^2}_{\substack{\text{(L,z+1)\&}\\ \text{(M,z+1)}\\ \text{are blocked}}} \underbrace{\gamma^2}_{\substack{\text{move}\\ \text{safely}\\ \text{into}\\ \text{(M,z) \&}\\ \text{(R,z+1)}}}$$

$$P_{EB}\{D>1\} = \underbrace{P\{D>1 \mid X=(L,z),\ \text{sq (M,z) is blocked}\}}_{\substack{\text{cond.}\\ \text{prob}\\ \text{given}\\ \text{sq (M,z)}\\ \text{is blocked}}} == \underbrace{P\{D>1 \mid X=(R,z),\ \text{sq (M,z) is blocked}\}}_{\substack{\text{cond.}\\ \text{prob}\\ \text{given}\\ \text{sq (M,z)}\\ \text{is blocked}}}$$

$$= \underbrace{\gamma}_{\substack{\text{sq}\\ \text{(L,z+1)}\\ \text{safely entered}}} + \underbrace{\beta\delta\gamma}_{\substack{\text{sq}\\ \text{(L,z+1)}\\ \text{blocked}\\ \text{safely enter}\\ \text{(M,z+1)}}}$$

$$P_M\{D>2\} = P\{D>2 \mid X=(M,z)\}$$

$$= \underbrace{\gamma}_{\substack{\text{enters}\\ \text{middle}\\ \text{sq. (M,z+1)}\\ \text{safely}}} P_M\{D>1\} + \underbrace{\beta\delta}_{\substack{\text{middle}\\ \text{sq.}\\ \text{(M,z+1)}\\ \text{block-}\\ \text{ed}}} \underbrace{\gamma}_{\substack{\text{safe}\\ \text{enter}\\ \text{sq to}\\ \text{left}\\ \text{(L,z+1)}}} P_{EB}\{D>1\} + \underbrace{(\beta\delta)^2}_{\substack{\text{mid}\\ \text{(M,z+1)}\\ \text{\& left}\\ \text{sq}\\ \text{(L,z+1)}\\ \text{blocked}}} \underbrace{\gamma}_{\substack{\text{safe}\\ \text{enter}\\ \text{next}\\ \text{sq (R,z+1)}\\ \text{on}\\ \text{rt}}} P_{EB}\{D>1\}$$

$$P_E\{D>2\} = P\{D>2 \mid X=(L,z),\ \text{sensor has not looked into (M,z)}\}$$

$$= \underbrace{\gamma}_{\substack{\text{safe}\\ \text{enter}\\ \text{sq}\\ \text{(L,z+1)}}} P_E\{D>1\} + \underbrace{\beta\delta\gamma}_{\substack{\text{sq on}\\ \text{edge (L,z+1)}\\ \text{blocked}\\ \text{safely}\\ \text{enter}\\ \text{middle}\\ \text{sq.(M,z+1)}}} P_M\{D>1\} + \underbrace{(\beta\delta)^2\gamma}_{\substack{\text{sq on}\\ \text{edge (L,z+1)}\\ \text{\&}\\ \text{middle(M,z+1)}\\ \text{sq}\\ \text{blocked}\\ \text{safe}\\ \text{enter}\\ \text{sq (M,z)}}} \underbrace{\gamma}_{\substack{\text{safely}\\ \text{enter}\\ \text{sq (R,z+1)}}} P_{EB}\{D>1\}$$

$$P_{EB}\{D>2\} = \underbrace{P\{D>2 \mid X=(L,z),\ \text{sq (M,z) is blocked}\}}_{\text{cond prob given sq. (M,z) is blocked}} = \underbrace{\gamma}_{\text{safe enter sq (L,z+1) on edge}} P_E\{D>1\} + \underbrace{\beta\delta\gamma}_{\text{sq on edge (L,z+1) blocked \& safely enter middle sq. (M,z+1)}} P_M\{D>1\}$$

$$P_M\{D>3\} = P\{D>3 \mid X=(M,z),\ \text{sensor has not looked at sq (M,z)}\}$$

$$= \underbrace{\gamma}_{\text{enters middle sq. (M,z+1) safely}} P_M\{D>2\} + \underbrace{\beta\delta}_{\text{middle sq. (M,z+1) blocked}} \underbrace{\gamma}_{\text{safe enter sq to left (L,z+1)}} P_{EB}\{D>2\}$$

$$+ \underbrace{(\beta\delta)^2}_{\text{middle sq (M,z+1) \& sq on left (L,z+1) blocked}} \underbrace{\gamma}_{\text{safe enter sq on rt (R,z+1)}} P_{EB}\{D>2\}$$

$$P_E\{D>3\} = P\{D>3 \mid X=(L,z),\ \text{sensor has not looked into sq (M,z)}\}$$

$$= \underbrace{\gamma}_{\text{enters edge sq. (L,z+1) safely}} P_E\{D>2\} + \underbrace{\beta\delta}_{\text{edge sq. (L,z+1) blocked}} \underbrace{\gamma}_{\text{safe enter middle sq to left (M,z+1)}} P_M\{D>2\}$$

$$+ \underbrace{(\beta\delta)^2}_{\text{middle sq (M,z+1) \& sq on left (L,z+1) blocked}} \underbrace{\gamma}_{\text{safe enter sq on rt (M,z)}} \underbrace{\gamma}_{\text{safe enter sq (R,z+1)}} P_{EB}\{D>2\}$$

$$\underbrace{P_{EB}\{D>3\}}_{\text{given } sq \text{ (M,z) is blocked}} = P\{D>3 \mid X=(L,z), \text{sq (M,z) is blocked}\}$$

$$= \underbrace{\gamma}_{\text{enters edge sq. (L,z+1) safely}} P_E\{D>2\} + \underbrace{\beta\delta}_{\text{edge sq. (L,z+1) blocked}} \underbrace{\gamma}_{\text{safe enter middle sq to left (M,z+1)}} P_M\{D>2\}$$

G. RECURSIVE FORMULA TO COMPUTE THE PROBABILITY THE SHIP CAN SAFELY CROSS THE FIELD

$$P_M\{D > d+1\} = P\{D > d+1 \mid X = (M,z)\} = \underbrace{\gamma}_{\substack{\text{enters}\\\text{middle}\\\text{sq. (M,z+1)}\\\text{safely}}} P_M\{D > d\} + \underbrace{\beta\delta}_{\substack{\text{middle}\\\text{sq.}\\\text{(M,z+1)}\\\text{block-}\\\text{ed}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter}\\\text{sq to}\\\text{left}\\\text{(L,z+1)}}} P_{EB}\{D > d\}$$

$$+ \underbrace{(\beta\delta)^2}_{\substack{\text{middle}\\\text{sq (M,z+1)}\\\&\\\text{sq on left}\\\text{(L,z+1)}\\\text{blocked}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter}\\\text{sq}\\\text{on rt}\\\text{(R,z+1)}}} P_{EB}\{D > d\}$$

$$P_E\{D > d+1\} = P\{D > d+1 \mid X = (L,z), \text{sensor has not looked into (M,z)}\}$$

$$= \underbrace{\gamma}_{\substack{\text{enters}\\\text{edge}\\\text{sq. (L,z+1)}\\\text{safely}}} P_E\{D > d\} + \underbrace{\beta\delta}_{\substack{\text{edge}\\\text{sq.}\\\text{(L,z+1)}\\\text{block-}\\\text{ed}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter middle}\\\text{sq to}\\\text{left}\\\text{(M,z+1)}}} P_M\{D > d\}$$

$$+ \underbrace{(\beta\delta)^2}_{\substack{\text{middle}\\\text{sq (M,z+1)}\\\&\\\text{sq on left}\\\text{(L,z+1)}\\\text{blocked}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter}\\\text{sq}\\\text{on rt}\\\text{(R,z)}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter}\\\text{(R,z+1)}}} P_{EB}\{D > d\}$$

$$\underbrace{P_{EB}\{D > d+1\}}_{\substack{\text{cond prob}\\\text{given } sq\\\text{(M,z)}\\\text{is blocked}}} = P\{D > d+1 \mid X = (L,z), (M,z) \text{ is blocked}\}$$

$$\underbrace{\gamma}_{\substack{\text{enters}\\\text{edge}\\\text{sq. (L,z+1)}\\\text{safely}}} P_E\{D > d\} + \underbrace{\beta\delta}_{\substack{\text{edge}\\\text{sq.}\\\text{(L,z+1)}\\\text{block-}\\\text{ed}}} \underbrace{\gamma}_{\substack{\text{safe}\\\text{enter middle}\\\text{sq to}\\\text{left}\\\text{(M,z+1)}}} P_M\{D > d\}$$

Note that

$$\underbrace{P_{EB}\{D > d+1\}}_{\substack{\text{cond prob} \\ \text{given } sq \\ \text{(M,z)} \\ \text{is blocked}}} = P\{D > d+1 \mid X = (R,z), (M,z) \text{ is blocked}\}$$

and

$$P_E\{D > d+1\} = P\{D > d+1 \mid X = (R,z), \text{sensor has not looked into (M,z)}\}$$

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